

The Institute of Informatics was founded in 1990. It consists of three departments and a research group:

- Department of Foundations of Computer Science
- Department of Computer Science
- Department of Applied Informatics
- Research Group on Artificial Intelligence

The Institute has a library which holds about 3000 Hungarian and English volumes and takes about 170 scientific journals. A scientific journal, *Acta Cybernetica* is published in English by the institute. The journal is available in about 150 university departments worldwide.

The main activity of the institute is education of modern informatics and computer science knowledge. The departments provide the introductory courses in informatics, programming, data processing and networks. These courses are available for all the students of the university. On the other hand, the institute offers B.S. and M.S. degrees in the following subjects:

- Computer Science (B.S.)
  - It takes 3 years.
  - The aim of this education is to teach the informatics experts who are in possession of up-to-date and high level informatics knowledge and the related foundations of mathematics.
- Computer Science (M.S.)
  - It takes 5 years.
  - The aim of this education is to teach the informatics experts who are in possession of up-to-date and high level computer science knowledge which are based on deep foundations of mathematics and computer science.
- Computer Science/Economics (M.S.)
  - It takes 5 years.
  - The aim of this education is to teach the informatics experts who are possession of up-to-date and high level informatics and economics knowledge which are based on deep foundations of mathematics.
- Education in Informatics
  - It takes 5 years.
  - The aim of this education is to teach the teachers for informatics who are in possession of up-to-date informatics and education knowledge and foundations of mathematics.

The curricula of the Departments of Informatics consist mainly of mandatory courses. The curricula have already been adjusted to the standards of leading universities, embracing most of the topics dealing with by a modern informatics and computer science

program, e.g. programming languages, compilers, operating systems, databases, networks, architectures, computer graphics, etc. There is a wide range of optional special courses available on the most up-to-date topics in computer science and informatics, e.g. parallel and distributed computing, expert systems, image processing, advanced operating systems, etc. There are also a number of basic mandatory courses, which place the emphasis on the theoretical aspects of computer science.

In addition to these programs, a doctoral program in Computer Science is offered since 1993. Since 2001 the Computer Science and the Mathematics Doctoral Schools formed a unified Doctoral School. The aim of our program is to support postgraduate computer science studies at the university, leading to the degree of Ph.D. in computer science, with emphasis on theoretical aspects. The possible research topics include mostly the parts of computer science and related areas, which are being investigated at the Departments of Informatics.

These programs and research tasks are supported by the scientific and educational cooperations (e.g. CEEPUS, SOCRATES / ERASMUS) with the following higher education institutes:

Boston University, USA,  
 Dresden University of Technology, Germany,  
 Friedrich-Alexander-Universität Erlangen-Nürnberg, Germany,  
 Kyoto Sangyo University, Japan,  
 Stevens Institute of Technology Hoboken, USA,  
 Technische Universität Graz, Austria,  
 Technische Universität Ilmenau, Germany,  
 Universidad de Almería, Spain,  
 Universität Bern, Switzerland,  
 Universität Hamburg, Germany,  
 Universität Karlsruhe, Germany,  
 Universität Stuttgart, Germany,  
 University of Nis, Yugoslavia,  
 University of Novi Sad, Yugoslavia,  
 University of Rome, Italy, and  
 University of Turku, Finland.

Among others, the following research areas are represented at the Departments of Informatics:

- Compositions of Automata
- Tree transducers and tree automata
- Global optimization
- Operations research
- Picture processing
- Artificial intelligence
- Term rewriting system

These research fields and the most important recent results of these topics are described shortly in the following. Further information is available at the web page of the institute: <http://www.inf.u-szeged.hu>.



# CURRENT RESEARCH & DEVELOPMENT IN SOFTWARE ENGINEERING

Árpád Beszédes, Rudolf Ferenc, Tibor Gyimóthy, and János Csirik

## I. TOPICS AND PEOPLE

The Software Engineering activity of the group involves several research areas as well as related industrial projects that have produced significant results. Most of the work is related to the *Reverse Engineering* of large software systems, particularly C++ programs. The reverse engineering activity deals more with the front-end parts of the process, such as the parser front-end and the higher-level understanding of object oriented systems (recognition of Design Patterns, like that below). Apart from these significant research is also being made in the field of *Program Slicing*, which is described below.

Finally, a new piece of research deals with the exploitation of relationships between XML documents and attribute grammars. This makes it possible to define XML attributes via *semantic rules*, making this novel approach suitable for compressing and understanding XML documents [9].

The software engineering activity involves several people having a variety of interests, including researchers and highly skilled programmers. This has resulted in the industry quality implementations of several research topics. In addition, several PhD and MSc students are also involved in a number of topics.

## II. REVERSE ENGINEERING

Most of the reverse engineering (RE) activity revolves around a reverse engineering tool developed under an industrial project with the Nokia Research Center in Finland. During this three-year project, a general RE framework was developed. In this work several research topics were studied that were related to reverse engineering: top-down parser implementation, graph drawing, abstracting higher-level models of programs, recognition of design patterns, etc.

### II.1. C++ REVERSE ENGINEERING

The developed RE framework is able to analyze large C/C++ projects and to extract their UML class model, as well as conventional call graphs. The main motivation for developing the system has been to create a general framework for combining a number of RE tasks and to provide a common interface for them. Hence it is a framework tool which supports project handling, data extraction, data representation, data storage, filtering, and visualization. All these basic tasks of the RE process for the specific needs have been accomplished by using the appropriate modules (plug-ins) of the system. Some of these plug-ins are present as basic parts of the tool, but the system can be extended to serve other reverse engineering requirements as well [5].

The parser of the tool is a command-line application that allows its integration into the user's makefiles and other configuration files. One of the greatest assets of the parser is probably the handling of templates and their instantiation at source level.

The C++ language processed by the analyzer meets the ISO/IEC standard from 1998. Furthermore, its grammar has been extended with the extensions used in Microsoft Visual C++ and Borland C++ Builder. The parser itself is a top-down parser generated by a parser generator tool and is based on LL(2) strategy augmented with ambiguity resolution mechanisms. The parser builds an internal representation of the program, which is then filtered and exported to various output formats such as a relational database, HTML report and an exchange schema as described below.

### II.2. C++ SCHEMA

One of the most important results of the reverse engineering project is that a *schema* has been elaborated to represent C++ programs. This schema is basically a model that describes how a C++ software system should be modeled using a reverse engineering tool. It also provides a common representation for the implementation, storage and exchange of a reverse-engineered software system. The C++ schema is given using UML class diagrams, which has several advantages. Firstly it uses a standard notation, secondly it is very close to implementation level, and thirdly it can be used as a basis for a programming interface for a tool that employs this schema. Externally the model of a software system using this schema can be stored in various physical formats, including XML-based graph representation. The schema has been successfully implemented and is used by other tools as well. In two recent papers [6, 3] we present our schema and provide guidelines for future work.

### II.3. RECOGNIZING DESIGN PATTERNS

The model of a C++ program that can be extracted via the reverse engineering tool can be used to recognize higher level relationships from the software (to address the recovery of design decisions). This goal is the aim of work, which tries to recognize standard Design Patterns which exist in the software, but which are not so explicitly expressed. The recognition is done by a separate tool that uses the output of the front-end.

The method consists of two separate phases; analysis and the reverse engineering of the C++ code, and architectural pattern matching over the reverse-engineered intermediate code representation. The pattern recognition effect is successfully realized by integrating two specialized software tools; our RE system and the architectural metrics analyzer *Maisa* (developed at the University of Helsinki). The method with the integrated power of the tool-set and initial results illustrated with small experiments are presented in [4].

### III. DYNAMIC SLICING

Program slicing is a powerful means for debugging, maintenance, reverse engineering and testing of programs. A slice consists of all statements and predicates that might affect a set of variables at a certain program point. A slice may be an executable program or a subset of the program code. More specifically, *dynamic* slicing methods compute those statements which influence the value of a variable occurrence for a specific program input. This means that dynamic slicing methods take advantage of dynamically available information during a program's execution.

An efficient algorithm has been elaborated for dynamic slicing, whose main advantage over prior methods is that it can be applied to real-size C programs because its memory requirements are proportional to the number of different memory locations used by the program (rather than the number of executed steps, which may be unbounded).

The presented algorithm is a forward global method for computing backward dynamic slices of C programs. In parallel to the program execution the algorithm determines the dynamic slices for any program instruction. The basic algorithm first appeared in [8], while in [1] we provide further experimental results and apply the algorithm to real C programs (this paper earned the title of the best paper of the conference). Another publication [2] deals with implementation details regarding the handling of unstructured statements by the algorithm.

### IV. CONFERENCES ORGANIZED

The seventh "Symposium on Programming Languages and Software Tools" (SPLST) [7] was organized by our Software Engineering group in Szeged in June 2001.

<http://rgai.inf.u-szeged.hu/~splst>

The scientific part of the next (sixth) "European Conference on Software Maintenance and Reengineering" (CSMR) will be organized by our Software Engineering group in March 2002.

<http://rgai.inf.u-szeged.hu/CSMR2002>

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This summary covers results achieved during 1998-2001 in the fields of developing nonlinear and global optimization procedures and in the investigation of their convergence properties. The majority of the studied algorithms are guaranteed reliability methods. This feature — not to be obtained for procedures based only on real arithmetic operations — was made possible by the use of interval arithmetic.

The basic problem is usually given in the form of

$$\min_{x \in X} f(x), \quad (1)$$

where  $f : \mathbb{R}^n \rightarrow \mathbb{R}$  is an (often two times continuously differentiable) real function, and  $X \subseteq \mathbb{R}^n$  is the set of feasible points. The set  $X$  is sometimes a simple  $n$ -dimensional interval, while in other cases it is determined by the constraints  $g_i(x) \leq 0$  and  $h_j(x) = 0$ . Global optimization deals with such solutions of problem (1) which are not only locally optimal, but where the related objective function has a globally minimal value.

The above problem class is very general, closely all mathematical problems can be reformulated as such nonlinear optimization problems. This is the reason why we cannot expect to have an algorithm that is capable to solve the whole problem class. Even for simple quadratic problems no polynomial solution algorithm exists. For general nonlinear problems the situation is even worse: such problems are not solvable at the costs of a finite number of function evaluations. The aim of the research is presently to widen the set of solvable problems with new algorithms, and on the other hand, to improve the reliability and the efficiency of the related methods.

The nonlinear and global optimization algorithms have a wide application area, let us just mention the system identification and parameter estimation problems often used in natural sciences. The completed procedures can be downloaded by anonymous ftp from

`ftp.jate.u-szeged.hu`

from the directory of

`/pub/math/optimization`

The clustering global optimization code available from here is downloaded 2-3 times daily, and in two independent comparison studies it proved to be the most efficient (“Of the programs tested, the Derivative-Free Boender-Timmer-Rinnooy Kan Algorithm by Tibor Csendes is the clear winner.” is written in <http://www.mat.univie.ac.at/~neum/glopt.html>, and see also the paper of Mongeau et al.<sup>1</sup>).

## I. INTERVAL METHODS FOR OPTIMIZATION

The largest set of results belongs to interval methods for global optimization. These techniques are based on

<sup>1</sup>Mongeau, M., H. Karsenty, V. Rouzé, J.-B. Hiriart-Urruty: Comparison of public-domain software for black-box global optimization. Optimization Methods & Software 13(2000) 203-226

a branch-and-bound algorithm, and the lower and upper bounds for the subproblems are calculated with either the interval arithmetic, or by more sophisticated inclusion functions based on it.

The present investigations aim to improve the efficiency of the guaranteed reliability algorithms, and in the same time enlarge the set of solvable problems. The first such task was to clear the role of multisection, i.e. the technique when not two (as in bisection), but a few more subintervals are produced in each iteration. The theoretical convergence results and the numerical testing was summarized in [8, 21].

In [16] we have investigated with an extensive a posteriori analysis which one-step-look-ahead decision rule would be the best to find out the multisection grade  $s$ . The answer is that the rule must produce the maximal possible difference in the inclusion function values for the resulting subintervals: the upper for one of the intervals must be as close to the lower bound of the other as possible.

Also the earlier mentioned multisection technique was improved by the newly found indicator, the so-called *Rejectindex*

$$pf(f^*, X) = \frac{f^* - \underline{F}(X)}{\overline{F}(X) - \underline{F}(X)}$$

first published by L.G. Casado and coworkers [2]. Its application ways to obtain better efficiency for other parts of the B&B method were discussed in [3, 12]. The latter paper is important, since it is the first successful approach since over 27 years to improve the interval selection step of the algorithm.

András Erik Csallner investigated the storage handling techniques of interval optimization methods in the paper [5], and the consequences of Lipschitz continuity for the stopping rules in [7]. He has successfully defended his PhD thesis [6], and has obtained the PhD degree with “Summa cum laude” in the year 2000.

The foundations of the interval methods and solutions to numerical problems were summarized in the papers [14, 15] (in Hungarian).

## II. CIRCLES PACKING IN THE UNIT SQUARE

The circles packing in the unit square problem is an old difficult field of computational geometry. It is also highlighted by the fact, that the ten circles case was first solved in 1990, and that no proven solutions exist above 27 circles (with the exception of  $n = 36$ ). Our first results, improving the earlier packings were presented 2000 in Mátraháza, Hungary at the Nonlinear Optimization Winter School, and will appear soon in [4, 25]. Further results are published by Péter Gábor Szabó in [23, 24], and the Japanese versions are covered together with other similar problems in [25]. The paper [20] by Mihály Csaba Markót discusses a verified algorithm for the same problem class. The comment [26] corrects an earlier paper on circles packing.

The improved packing for 47 circles is demonstrated on Figure 1.



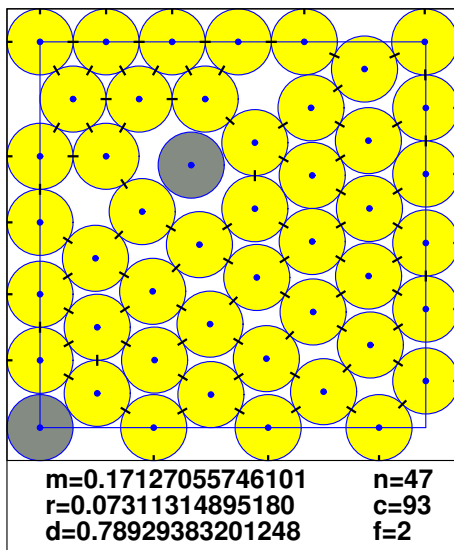


Figure 1. The found improved packing of 47 circles in the unit square. Here  $r$  is the common radius of the circles,  $d$  is the density of the packing,  $c$  stands for the number of connections between circles or between a circle and a side, and  $f$  denotes the number of free circles.

### III. OTHER OPTIMIZATION RELATED RESEARCH

János Balogh studied stochastic global optimization techniques applied successfully to a chemical phase stability problem in [1]. The same author has published a paper on a parallelized version of a random search method for global optimization with coauthors [22]. Boglárka Tóth and Tamás Vinkó tested a sophisticated interval arithmetic based algorithm on standard test problems. The results are published in [27].

The KÉSZ Ltd. asked us to contribute to its production control system with a cutting algorithm that improves the cutting of iron rods used in buildings. The research ended in the suggestion of a compound algorithm containing packing heuristics and enumeration procedures [17, 18] allowing a few percentages of savings in a very high yearly raw material budget.

The possible application of stochastic and interval optimization techniques in chemical process network design problems were discussed in [9].

### IV. RESEARCH PROJECTS

Here we summarize the research and development projects of the last years won. In most cases Tibor Csendes was the (Hungarian) project head (other cases are marked). For each project the source, identification number, time period, project title, and number of participating researchers are given.

**OTKA** T 016413, 1995-98, "Nonlinear optimization and decision support", 14 persons, (head of the project: Rapcsák Tamás).

**OTKA** T 017241, 1995-98, "Reliable methods in global optimization", 6 persons.

**OMFB** Hungarian-German Bilateral, D-7/97, 1998-2000, "Interval arithmetic based global optimization", 4 persons.

**FKFP** 0449-99, 1999-2000, "New multisection techniques in global optimization", 2 persons.

**OTKA** F 025743, 1998-2001, "Optimization techniques", 2 persons (head of the project: András Erik Csallner).

**OTKA** T 032118, 2000-2003, "Efficient interval algorithms in nonlinear optimization", 1 person.

**MTA** Hungarian-Bulgarian collaboration, BGA-20, 2000-2002, "Optimization in chemical phase equilibrium problems", 4 persons.

**OTKA** T 034350, 2001-2004, "Developments of global optimization procedures", 8 persons.

**OMFB** Hungarian-German Bilateral, D-30/00, 2001-2003, "Global optimization procedures", 7 persons.

**MÖB-DAAD** Hungarian-German, 11/01, "Application of global optimization methods in approximation problems", 7 persons.

**KÉSZ** Ltd. Szeged, "A cutting problem in metal industry", 2 persons.

**OMFB** Hungarian-Spanish Bilateral, SP-25/01, 2001-2003, "Reliable methods of global optimization and their parallel implementation", 11 persons.

### V. CONFERENCES ORGANIZED

The authors organized four conferences in the interval 1998 – 2001. The first two are meant as the possibly first conference for many PhD students of computer science, it is the Conference for PhD Students in Computer Science ( $CS^2$ ) held in Szeged, Hungary, 18-22 July 1998 and 20-23 July, 2000. There were well over 60 accepted talks from many countries, and around 70 participants each time. It is hoped that this conference will be repeated either in Szeged again, or by one of our collaboration partners. The Acta Cybernetica, and in the second time also the Periodica Polytechnica devoted a special issue to the fully refereed papers emerging from the talks. The related information together with the lists of accepted papers and coordinates of the participants can be found at

<http://www.inf.u-szeged.hu/~cscs>

The SCAN-xx conference series covers scientific computing, validated numerics and computer arithmetic related fields. The SCAN-98 was organized in Budapest, Hungary, 22-25 September, 1998. Since the main research interest of the authors belong to this discipline, it is a big honour that they were asked and accepted to have the responsibility to organize the next meeting. It was the first time that this conference is organized in Hungary. Over 90 papers were submitted from 20 countries, among them exotic like Jordan, India or Brazil. There were about 70 talks held and over 100 participants. The fully reviewed papers arising from the talks appeared in a special issues of the main journal of the field, Reliable Computing [11], and in an edited volume published by Kluwer Academic Publisher [10]. More details are to be read at

<http://www.inf.u-szeged.hu/~scan98>

In the case of the forth conference, Tibor Csendes was a member of the Organizing Committee of the EURO XVII conference, the main meeting of the European researchers in operations research. It was organized in Budapest, Hungary, July 16-19, 2000. The details can be found at

<http://www.sztaki.hu/conferences/euro17/>

Tibor Csendes edited the special issue of the Central European Journal of Operations Research devoted to the papers emerging from the talks of the Hungarian Operations Research Conference held in Veszprém, 1999 [19] with Tamás Rapcsák.

## VI. CONNECTIONS OF RESEARCH TO EDUCATION

The above research relates mostly to the Global Optimization, Nonlinear Optimization and to the Interval Mathematics special courses and is also connected to the Global Optimization PhD course. The open problems of some particular fields can serve as research subjects for student research competition (TDK) or for PhD students.

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In the last few years both the computers and the softwares have changed a lot. Nowadays there is an information overload. We can say that the amount of information stored in computers doubles in every year. So there is a great demand for programs reducing these huge data masses to usable and understandable quantities, helping in discovering relations not only for experts, but everyday people too.

Until now only artificial intelligence has been dealing with realizing human tasks for computer. In the 90s appeared the so-called computational intelligence, which aimed at mathematical modeling of human operations between perception and logical reasoning. They gave new algorithms and methods on the area of multi-criteria decision making, genetic algorithms and fuzzy sets.

### I. MEMBERSHIP FUNCTION

Fuzzy sets realize the formal description of the human non-exact concepts. According to the earlier interpretations the membership function is the modeling of the uncertainty. But the successful practical applications do not use uncertainty. We showed that using elementary preference function (which is the approximation of the characterization function) instead of the membership function can solve this problem.

The proposed function basically represents the intensity of preference (for example smaller, better, more favourable soft estimation). This is a logical function, which has got the following parameters: expectation level, decision threshold and the sharpness of the preference.

We showed, that the membership functions used in the fuzzy theory can always be expressed as composition of two functions. The first is the mapping of the perception to a scale, the second is the earlier mentioned elementary preference function. So, the subjective factor, which is not the subject of mathematical disciplines (sociology, psychology), can be separated from the fuzzy theory, which makes possible to build a formal and theoretically consistent system.

### II. CONNECTIVE OPERATORS AND THE NEGATION

Another critical point of the fuzzy theory is the choice of continuous logic operators. The number of the introduced "and" and "or" operators have been growing in the last few years. More and more new operators had been introduced beside the first "min" and "max" operators. General description of these operators is given by the t-norms and t-conorms. T-norms is the family of monotonous operators fulfilling the associativity.

Its main classes are the following: strictly monotonous operators (if there is no idempotent element inside the domain of the operator), nilpotent operators, min-max operators and drastic operators. If the associativity is not assumed, the number of operators is much greater. Their full description is given by solutions of associative functional equation and representations of ordered semigroups.

One aim of the researchers is finding these kinds of operators. Another task is to find reductions of the mentioned family of operators, to find those representations which are the best in the aspect of practical application. Further restriction is given by the consistency of the system. In case of strictly monotonous operators the necessary and sufficient conditions of DeMorgan classes are also the result of the latest research. Applying this theory we can also get the representation of the negations. Until now in the class of strictly monotonous connective operators and the negation were not in any relation. The negation is not unique. Its parameter is the neutral value (the fix point of the negation), which can be interpreted as a decision level.

### III. UNARY OPERATORS

In the fuzzy theory beside the negation there were also a lack of the mathematical basis of the other unary operators. We still use the modifiers (hedges) defined by Zadeh in a heuristic way. As the applied hedges are independent from the conjunctive and disjunctive operators, the transformation is always the same.

The consequence is that the fuzzy theory became more and more sophisticated. Unary operators could be defined to be consistent to the other operators. The negation – which has connection to the other logic operators by the DeMorgan law – helps us defining unary operators as composition of two negations with different neutral values. These unary operators are unique up to isomorphic. Functions of the isomorphic mappings are the same as the generator functions of the operators that come from the solutions of the associative functional equation.

Using the logic operators we have to deal with decision levels. Using unary operators the different decision levels can be transformed to the "same bases", they could be regarded as idempotent values. In this way an operator can be derived which is a special case of general mean-values when all the weights are the same.

### IV. MULTI-CRITERIA DECISIONS AND OPERATORS

At multi-criteria decisions we have to deal with the importance of the certain features through the evaluation. We showed, that the operators can be generalized further keeping their earlier features. The weighting method is an operator dependent transformation and it can be linearized on the weights, which increase the practical applicability of such multi-criteria decisions.

The earlier studied aggregation can be connected with the logical operators, because it comes from the lines of weighted logical operators. A nice feature of these operators is that we can get back the conjunction, disjunction, aggregation, min max operator, if the parameter of the operator is  $-\infty, -1, 0, +1, +\infty$ , so by the change of just one parameter we can get the whole spectrum of the operators. By the help of aggregation another unary operator can be made,

which transform the sharpness. Preference operator can be made from aggregation by the help of negation. Decisions based on preference and utility theory are equivalent if we apply the preference coming from aggregation operator.

## V. FLEXIBLE (PLIANT) SYSTEM

There are several approaches to treat databases, like

- classical analysis: probability theory, statistical analysis (too technical, so the everyday people cannot use it)
- new algorithms: neural networks, genetic algorithms (more effective and a bit more user-friendly, but the success depends on the settings of the technical parameters made by the user)

As the classical mathematical logic is different from the everyday used logic, there is a need of introducing a new concept. Pliant (flexible) system can handle these concepts in a unified way, and by applying it reasoning can be made.

Pliant system contains logical elements and decision elements, like aggregation (which is a special summary of data features and it is very important in decision making) and preference modeling. It can integrate the often-used fuzzy systems, neural networks and decision trees, i.e. this logic gives a common base of paradigms developed in the last few years. It can also be applied in clustering methods and essentially improves the technologies based on fuzzy control. A brand new class of learning algorithms can be derived from the system.

We can get pliant system if we use Dombi formula instead of general operator. This operator system is isomorphic with the general one. We can see that in pliant system the preference operator and the interpretation of the membership function by sigmoid function is consistent. The nilpotent operator class can be built up similar like the earlier mentioned monotonous operator class. We can differ two kinds of applications: soft computing methods and decision support methods. Applying this system we can make operations on numbers and the result should be composed in natural language ensuring the easy understanding.

## VI. GENETIC ALGORITHMS

Genetic algorithm is a stochastic searching method based on the modeling of evolution. We have introduced concepts which were present in the literature on an intuitive way. Starting from the theoretical questions, introducing the concept of species, using a new method of localization of competition and reproduction we made GAS suitable for discovering the local optimum of functions. The developed program called GAS realized the applicability of the theory.

The aim of the further researches was to choose the suitable diameter of species. In spite of the earlier theories this value was provided by a function while the algorithm was running, so the value was continuously decreasing. The theoretical base of this concept can be considered as an important step on this research area.

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## I. ITERATION THEORIES AND THE AXIOMATIZATION OF THE FIXED POINT OPERATION IN COMPUTER SCIENCE

The purpose of the work carried out from 1998 to 2001 was to obtain complete, but simple, descriptions of the properties of the fixed point or iteration operation in computation. Previous work by Bloom, Ésik and others has resulted in a complete description of the equational laws satisfied by the iteration operation. This description takes the form of the axioms for *Iteration Theories* [6, 7], certain (enriched) Lawvere algebraic theories, which capture important features of many classes of structures of interest in the theory of computation involving ordered and metric structures, functors and 2-categories, trees and synchronization trees, and others. In [36], we have proved that functor theories over algebraically complete categories, where iteration is defined by initial algebras, result in iteration theories. The importance of algebraically complete categories for theoretical computer science is that these are exactly those categories in which one can define abstract data types in a canonical way. In [10], we have introduced iteration 2-theories that are a 2-categorical generalization of iteration theories. We have described the structure of the free iteration 2-theories.

One of the axioms of iteration theories, the commutative identity, was quite complex and difficult to verify in particular cases. In [21], we have replaced this axiom by the *group-identities* associated with the finite (simple) groups. The completeness of the group-identities confirms a classic conjecture of Conway in an extremely general setting. Refinements and further analysis of this result were obtained in [20, 23]. In [9], we have shown that iteration theories cannot be axiomatized by a finite number of equation schemes. Moreover, in [24], we have described all iteration theories of boolean functions. In the paper [5], we have described the structure of the free Conway theories.

## II. APPLICATIONS OF THE EQUATIONAL LOGIC OF FIXED POINTS

As major beneficial corollaries of the general theory of fixed points, we have obtained relatively simple sets of equational and implicational axioms for the binary supremum operation on continuous functions [25], the concurrent behavior of processes with respect to several behavioral equivalences [25, 29], regular tree languages [17]. For an application to Lindenmayerian power series, see [33]. For a survey, see [28].

## III. AXIOMATIZING REGULAR LANGUAGES AND RATIONAL POWER SERIES

In [13], we solved an open problem of A. Salomaa by showing that the equational theory of commutative regular languages (commutative Kleene algebra) is the same as the theory of one letter regular languages. Thus, since the theory of commutative regular languages is nonfinitely based, so is the theory

of one letter regular languages. In [15], we proved that the equational theory of regular languages with concatenation and Kleene star is nonfinitely based. This solves a problem of Bredikhin. In [34], we extended Kozen's axiomatization of the equational theory of regular languages to rational power series with coefficients in any finite commutative ordered semiring. In [14], we proved that the equational theory of Kleene algebras of binary relations with relational inverse is not finitely based.

## IV. TROPICAL SEMIRINGS

In a series of papers, we studied the equational theory of the tropical semirings. We established several axiomatizability and decidability results and described the structure of the free algebras in the corresponding varieties. See [4, 1, 2, 3].

## V. THE SHUFFLE OPERATION AND POMSET MODELS OF CONCURRENCY

In a series of papers, we extended the study of the equational theory of the regular operations on languages to include the shuffle operation which, in the interleaving model of concurrency, is used to model parallel execution of processes. We have shown that the equational theory of the shuffle operation is intimately related to the theory of pomsets equipped with variants of the series and parallel composition operations. See [38, 32, 30]. In [39], we solved an open problem of J.-E. Pin.

## VI. COMPOSITIONS OF AUTOMATA

In [22], we gave a simple proof of the Krohn-Rhodes decomposition theorem for finite automata. In [16], we have shown that there is a Letichevsky automaton not complete with respect to homomorphic representation by the  $\nu_2$ -product. (On the other hand, it is known that every Letichevsky automaton is complete with respect to homomorphic representation by the  $\nu_3$ -product.)

## VII. GENERALIZATIONS OF AUTOMATA

In [19], we defined the syntactic theory of a regular tree language as the minimal theory that recognizes the tree language. Using this concept, we gave an extension of Eilenberg's Variety Theorem to regular tree languages. This variety theorem establishes a bijective correspondence between classes of regular tree languages closed with respect to the Boolean operations, inverse morphic images and quotients, and suitably defined varieties of algebraic theories. In [37], we studied automata on higher dimensional words and extended several basic results of the theory of automata to this setting.

## VIII. ALGEBRAS FOR ASYNCHRONOUS CIRCUITS

We have proposed a family of algebras to study the hazards of asynchronous circuits, cf. [11, 12]. These algebras are commutative De Morgan bisemigroups and are a generalization ternary algebras. In [27], we gave a geometric description of the free (commutative) De Morgan bisemigroups and De Morgan bisemilattices. In [18], we showed that every ternary algebra can be faithfully represented by ternary algebras of functions.

## IX. CONFERENCES AND EDITORIAL WORK

In addition to a large number of other conference presentations, Zoltán Ésik was invited speaker at the following conferences: Words, Languages and Combinatorics (Kyoto, 2000), Category Theory (Como, 2000), Developments in Language Theory (Vienna, 2001), Fixed Points in Computer Science (Florence, 2001), Workshop on Max-Plus Algebras (Prague, 2001), Expressiveness in Concurrency (Aalborg, 2001), 6th Int. Conf. Discrete Mathematics and Applications (Banská, 2001). He served on the steering committee of the Fundamentals of Computer Science conferences and on the program committee of the conferences Universal Machines and Computations (Metz, 1998), Foundations of Software Science and Computation Structures (Lisbon, 1998), Fixed Points in Computer Science (Brno, 1998), Automata and Formal Languages (Vasszéchény, 1999), Fixed Points in Computer Science (Paris, 2000), Fixed Points in Computer Science (Florence, 2001). Zoltán Ésik edited the proceedings of FICS 98 and served as an editor of the journals Theoretical Computer Science, Theoretical Informatics and Applications, Discrete Mathematics and Theoretical Computer Science, Acta Cybernetica, Acta Sci. Math. He held visiting positions in Aizu, Waterloo and Aalborg.

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# ANALYSIS OF MEDICAL IMAGES AND NETWORK COMMUNICATION

Márta Fidrich

This summary covers results achieved during 1998 - 2001 in two fields: medical image processing and network communication. These fields are absolutely distinct, the reason to write one common summary about them is that the principal investigator of the two topics is the same person.

## I. ANALYSIS OF MEDICAL IMAGES

I started to study medical images in scale space during my PhD at INRIA in France. I applied this knowledge to analyse facial laser scans while I was a research fellow in Leeds, and this led to a successful collaboration between the Department of Informatics at the University of Szeged and the Centre of Medical Imaging Research at Leeds.

### I.1. Scale-space theory

An image contains huge amount of data, where the relevant information often remains hidden. Similarly, human brain gets enormous visual information through the retina, and only uses a fraction of it. It can be supposed that a multiscale, pre-processing and selective mechanism works in our visual front-end. The scale-space theory aims to apply the model of *human vision* in image processing, so that the analysis of images become automatic. Another approach to scale-space theory emphasizes the role of observation scale range, well known in physics: Real objects are observed at a range of *measurement scales* (cell - leaf - tree - forest) in contrast to ideal mathematical entities (point, line). (A summary about scale space is provided in [1].)

To study an image at multiple scales, the following steps are needed:

- Embed the original image  $f(\mathbf{r})$  into a continuous family of images  $\phi(\mathbf{r}, t)$  so that  $\phi(\mathbf{r}, 0) = f(\mathbf{r})$ , where  $t$  is the scale (resolution) and  $\mathbf{r}$  is a position vector. As scale increases, images should be simplified (no “spurious details” generated); the image size does not change (no spatial sampling).
- Analyse the image family (relations between image structures) in function of increasing  $t$ .
- Build a model, which can be used in practice.

The homogenous *linear scale space* is the most studied and understood example for scale spaces. It is a mathematical formulation of “we do not know anything, we do not have any preferences”, a presumable model of the first phase of human vision. Although, it can be axiomatized in several ways, it can only be created by the convolution with a Gaussian of increasing variance. If a feedback is added to the previous model, that is, we let diffusion depend on local quantitative image characteristics (gradient, curvature), various *non-linear scale spaces* can be created. We note that our vision system also uses feedback, when we focus on an interesting detail.

An image in linear scale space is simplified as scale increases: noise and small-scale features disappear,

only the main structures remain. Unfortunately, images become blurred, even at places where it is not requested (at edges). On the other hand, this scale space has a vital role to *extract image features correctly*. We analysed features (such as high-curvature corner points, ridges), defined by differential invariants, in 2D and 3D at multiple extraction scales. In particular, we analysed the stability of corner points in this scale space [2], we assessed their significance based on scale-related properties and we compared the usefulness of different stability criteria for registration.

It is well-known, that the position of image features may change as scale increases. Thus *features have to be detected at a coarse scale then followed to the original fine scale*. This is rather difficult in practice, since both spatial position and scale are discretized. I developed a new method to follow features efficiently across scale, based on the concept of intersecting iso-surfaces. To do so, I introduced a novel algorithm, which allows us to search for iso-surfaces and their intersections in any dimension. This nD algorithm is based on new orientation and (de-)composition considerations, thus the reconstructed iso-surfaces and their intersections have good topological properties, which can be easily proved, moreover the implementation is quite straightforward. This methodology was applied to medical image processing such as automatic labelling of organs, matching and craniofacial reconstruction. For a description, see [3].

### I.2. Analysis of facial growth

Clinical *growth studies* are becoming increasingly important for assessing whether the change of shape related to age is normal i.e. to see whether the shape measurements are within an accepted range (variance) from the average values. Our primary interest was to study growing faces of individuals (versus cross-sectional data) in order to compare the development of control subjects with subjects having problems with asymmetry (hemifacial microsomia) or orthodontics. To measure facial dimensions, the optical *laser scanner* is used. The clinical interest was the ability to predict the long-term effect of surgical treatments which would help doctors to decide whether a specific problem should be over- or under-corrected so that the long-term result would correspond as much as possible to the desired one.

I organised and coherently represented data of facial laser scans taken in a 10-year period. I developed software for the visualisation and subsequent analysis of data: identification of anatomical landmarks and registration of facial scans at different ages. After these prerequisite steps, I began to work on the statistical description of the data, visual perception of age and modelling shape change during growth. First results can be found in our study report [4], which was followed by specialized statistical analysis of facial growth, see [5, 6].

### I.3. Robust surface matching

A necessary tool to detect changes due only (mostly) to growth is a suitable *registration algorithm*. It must optimally match the foreheads while emphasizing the differences in the lower parts of the faces. (Doctors agree that growth of the forehead finishes at about 5 years of age, while the rest of the face keeps growing until young adulthood.) Practitioners often use only manual registration techniques based on anatomical landmarks and their analysis is naturally dependent on the accuracy of the alignment of faces. This alignment can be further improved by developing an accurate automated registration algorithm.

There is a vast literature on registration of grayscale datasets. However, these algorithms could not be applied to our task, which needed specialized methodologies. Indeed, changes between facial scans could be of the order of centimetres rather than sub-millimetres: a significant proportion of the surface may be quite different due to surgical intervention or growth. Further, regions of the surface to be matched may be very similar, leading to a challenge analogous to that of aligning two spheres.

That is why we developed a technique based on iteratively matching surfaces using information about surface curvature. What makes our algorithm accurate and robust is a new way to find correspondence between point sets: CSM. A robust Corresponding point is calculated by determining the Sensitivity of a correspondence to Movement of the point being matched. If the correspondence is reliable, a perturbation in the position of this point should not result in a large movement. A measure of reliability is also calculated. The correspondence calculation method is independent of the (iterative) registration algorithm, their combination provides very good results. We have several papers describing our method [7, 8, 9].

## II. NETWORK COMMUNICATION

Research related to communication and network technologies has started only in 1999 at the Departments of Informatics under my direction. Our topics focus on the changeover of the networking protocol of the Internet that should take place soon. We have examined the impact of the deployment of IPv6, such as how the changeover can be solved in practice, what the expected results are, and what the possible problems can be. We have a public web page, <http://nokia5.inf.u-szeged.hu/pubpage/pubhp.html> containing information on our projects.

### II.1. Comparison of IPv4 and IPv6

The Internet Protocol version 4 (IPv4) has been available since 1981. Although its success is indisputable, the rapid growth of the Internet has created a number of problems for the administration and operation of the global network. Several features of IPv4 need improvement, such as too short address length, ponderous routing mechanism, variable-length header options and laboring configuration of hosts. Some new challenges that must be handled have also appeared, such as security, mobility and real-time support. So that the Internet could continue its remarkable growth, all these problems have to be solved. Clearly, patching of IPv4 is not sufficient, a new protocol is needed.

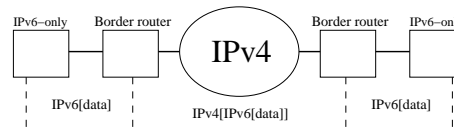


Figure 1. Tunneling mechanisms.

The new version 6 of the Internet Protocol (IPv6) was specified in 1995 and it has been finalized after years of debate. It includes expanded addressing capabilities, header format simplification, improved support for extensions and options, plug and play services, authentication and privacy capabilities, native mobility support, and also real-time and quality services. Presently the transition is underway, which would take still some years and will possibly raise new questions to be replied and new problems to be solved.

We wrote a comprehensive document [10] about the two network layer protocols: IPv4 and IPv6. We have pointed to the limits of IPv4 and compared the two protocols from several aspects (base functionality, addressing & routing, neighbor discovery & autoconfiguration, security, mobility, realtime support & quality of service). A reference table is also provided, which lists similarities and major differences between IPv4 and IPv6, thus helps to understand where, why and how IPv4 needs to be changed. We also paid special attention to the transition from IPv4 to IPv6 and to the problems caused by the future co-existence of these incompatible protocols (possibility of interwork, routing aspects & practice, implementation issues). This handbook is composed of four chapters and an appendix, and suggested to be used for teaching purposes.

### II.2. Inter-operation mechanisms

To facilitate the long process of changeover, several inter-working and transition mechanisms were designed between IPv4 and IPv6, falling into two categories. The first case is when we want to run an IPv6 network using the existing infrastructure i.e. IPv4 network; see Fig 1. Depending on the mode and pre-conditions of use of IPv4 network, three main types of solutions are distinguished: (1) IPv6 over IPv4 *tunneling*, which can be automatic or configured, (2) IPv6 over IPv4 without explicit tunnels (*6over4*) (3) connection of IPv6 domains via IPv4 clouds without explicit tunnels (*6to4*).

The second case is when IPv4-only and IPv6-only hosts want to communicate with each other. This communication is very complicated because of the different protocols (e.g. addressing schemes, protocol headers and fragmentation). The communication can be done at one of three different protocol levels. Header conversion (example: NAT-PT employing SIIT *translator*) is used at the IP level; see Fig 2. Relaying of UDP and TCP sessions (example: SOCKS) can be done at the transport level, while application proxies (such as http, gopher, pop3, telnet, ftp proxy) mediate at the application level.

Before buckling to the overall change of the installed protocols, it is worth analyzing and comparing these mechanisms – and that motivated our next study. We tested and compared configured tunneling, 6to4 and SIIT, against plain IPv4 and IPv6 sending and receive



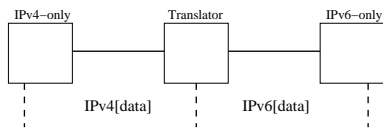


Figure 2. Protocol translation using SIIT.

ing. As far as we know, we are the first who comparatively analysed inter-operation mechanisms.

We had two aims concerning the tests: *algorithm-test* shows the overhead caused by the additional packet processing steps of the selected techniques against plain IPv4 and IPv6 mechanisms; while *router-test* indicates how fast the machine works as an Internet router and how big its packet throughput is. It turned out that available test tools are not suitable for our purposes, because they do not support IPv6 or they are not designed for tests within one single host. Thus we developed our own testbed, which turned out to be a very flexible and easy-to-use tool.

We presented our measurement results with detailed explication. The interested reader is referred to [11] and [12]; the latter is an extended version containing more tests as well thorough interpretation and discussion of the results.

#### New communication environment

The 3G (Third Generation) Mobile Networks use SIP (Session Initiation Protocol) as their call control protocol and IPv6 as network layer protocol for wireless communication. When IPv6 was chosen for 3G networks, an urgent demand appeared in connecting the existing IPv4-based networks to the new (IPv6) one. Although IPv6 was designed to extend IPv4, the extension incorporates so many fundamental changes, that IPv6 is not compatible with IPv4: there is no way of direct communication between IPv4-only and IPv6-only systems. On the other hand, programs in both realm do have to communicate with each other. Since IP addresses may also be important information carried in application layer protocols, (FTP, SIP and SDP), IP-level gateways only are not enough for successful communication: gateways at different levels (network, transport and application) have to be developed.

The goal of our following work was to create a demonstration system established on a novel way of connecting 3G mobile networks based on IPv6 and legacy Internet phones based on IPv4. As far as we know, we are the first, who analysed and developed such a communication system.

The main idea was to connect a mobile SIP User Agent, based on IPv6, and another SIP User Agent, based on IPv4, via two SIP proxies (IPv6 and IPv4) and NAPT-PT (Network Address Port Translator - Protocol Translator). Indeed, the real SIP communication between the IPv4 and the IPv6 networks means the communication between the two proxies via the NAPT-PT. When a media connection is created, the IPv6 SIP proxy (Media Gateway Controller) sends a command using the MEGACO (Media Gateway COntrol) protocol to add a binding to the NAPT-PT (Media Gateway). This binding helps the NAPT-PT to convert the packets transporting the media connection. When the media connection is terminated, the IPv6 SIP proxy sends a message to the NAPT-PT to release the binding.

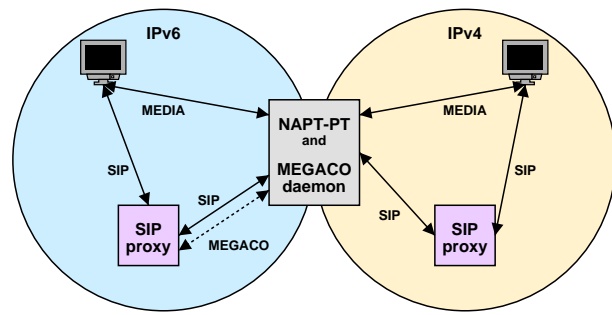


Figure 3. Test environment for multimedia call initiation between IPv4 and IPv6

The communication environment presented in Figure 3 requires the development of three entities making a fairly complex system. Our software engineering task was not usual: we had to develop parts from kernel to user level, from network to application layer of the TCP/IP protocol family, in C and C++ languages. In addition, we had to make a parser for the MEGACO protocol (having highly complex grammar). Indeed, communication of our software pieces required novel solutions and expert techniques; we could also make good use of various development tools. The interested reader is referred to [13] for more details.

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The list of participants is given below in alphabetical order, for their contribution see the publications. Bátri, Dénes; Bohus, Mihály; Dikán, Gábor; Fidrich, Márta; Fóris, Gábor; Hendlein, Péter; Kiss, Gergő; Martonossy, László; Notaisz, Krisztián; Sebő, Marianna; Sógor, László; Somlai, Gábor; Tarjányi, Tamás.

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## I. TREE TRANSDUCERS

In [1] we made a comprehensive study of top-down tree transducers, attributed tree transducers, and macro tree transducers. The previous results are summarized and discussed in a unified terminology. Moreover, some new results are presented mainly concerning the comparison of the computation power of the different tree transducer models.

It is known for a long time that macro tree transducers are more powerful than attributed tree transducers. In [5] we give a characterization of the class of tree transformations which are computed by (non circular) attributed tree transducers, in terms of so called attributed-like macro tree transducers. Moreover, we prove that it is decidable whether a macro tree transducer is attributed-like.

In [6] we prove that the domains of partial attributed tree transducers are the tree languages recognized by tree walking automata in universal acceptance mode. Moreover, we prove that the domains of partial attributed tree transducers having monadic output are the tree languages recognized by deterministic tree walking automata.

In [8] we consider attributed tree transducers of type simple multi-visit, multi-sweep, multi-alternating pass, and multi-pass following the definition of the same subclasses of attribute grammars. We characterize the tree transformation classes induced by the considered four subclasses of attributed tree transducers in terms of macro tree transducers. Namely, we define simple multi-visit, multi-sweep, multi-alternating pass and multi-pass macro tree transducers and prove formally that the attributed tree transducers and the macro tree transducers of the same type induce the same tree transformation class. Also we give an inclusion diagram of the tree transformation classes induced by the above and some further fundamental types of attributed and of macro tree transducers.

In [10] we consider the closure  $UCI(Rel)$  of the class of relabeling tree transformations, under  $U$ =union,  $C$ =composition and  $I$ =iteration. We give a characterization of  $UCI(Rel)$  in terms of a short expression built up from  $Rel$  with composition and iteration. We also give a characterization of  $UCI(Rel)$  in terms of one-step rewrite relations of very simple term rewrite systems. We give a similar characterization of  $UC(FRel_+)$  where  $FRel_+$  is the class consisting of the transitive closures of all functional relabeling tree transformations. Finally we show that  $UCI(Rel) = UCI(FRel)$ .

In [11] we generalize bottom-up tree transducers and top-down tree transducers to the concept of *bottom-up tree series transducer* and *top-down tree series transducer*, respectively, by allowing formal tree series as output rather than trees, where a formal tree series is a mapping from output trees to some semiring. We associate two semantics with a tree series transducer: a mapping which transforms trees into tree series (for short: tree to tree series transformation or t-ts transformation), and a mapping which transforms tree series into tree series (for short: tree series transformation or ts-ts transformation).

We show that the standard case of tree transducers is reobtained by choosing the boolean semiring under the t-ts semantics. Moreover, we show that certain fundamental constructions and results concerning bottom-up and top-down tree transducers can be generalized for the corresponding tree series transducers. Among others, we prove that polynomial bottom-up t-ts transformations can be characterized by the composition of finite state relabeling t-ts transformations and boolean homomorphism t-ts transformations. Moreover, we prove that every deterministic top-down t-ts transformation can be characterized by the composition of a boolean homomorphism t-ts transformation and a deterministic linear top-down t-ts transformation. We prove that deterministic top-down t-ts transformations are closed under right composition with nondeleting and linear deterministic top-down t-ts transformations and are closed under left composition with boolean and total deterministic top-down t-ts transformations. Finally we show that nondeleting linear bottom-up and nondeleting linear top-down tree series transducers generate the same t-ts transformation class.

## II. COOPERATING DISTRIBUTED TREE AUTOMATA AND TRANSDUCERS

In [4] distributed tree processing devices, like regular tree grammars, top-down tree automata and transducers, and bottom-up tree automata and transducers are considered. The concept of distribution lies in that the set of rewriting rules are distributed among  $n$  sets, called components. The components work on deriving a sentential form such that they cooperate with each other concerning cooperation strategies. We develop a technical toolkit for studying distributed tree processing devices. We mainly consider the  $\dashv$  strategy which means that a component should work on a sentential form as far as it can. We show that in general the above distributed tree processing devices with  $\dashv$  cooperation strategy are more powerful than ordinary ones with respect to generating capacity.

In [2] we prove that the class  $ETOLT$ , the class of tree languages generated by ETOL tree systems, can be characterized in the following way:  $ETOLT$  is equal to the class of tree languages generated by distributed regular tree grammars cooperating with termination strategy [4]. This result is a generalization of the corresponding one for ETOL systems and distributed context-free (string) grammars cooperating with terminal strategy.

In [9] we show that the component hierarchy of chain-free distributed regular tree grammars cooperating with terminal strategy [4] is infinite with respect to tree language generating capacity. More exactly, we prove that  $cfCDRTG_{\dashv}(n) \subset cfCDRTG_{\dashv}(2(n-1)^2+3)$ , where  $n \geq 1$  and  $cfCDRTG_{\dashv}(n)$  denotes the class of tree languages generated by chain-free distributed regular tree grammars of at most  $n$  components cooperating with terminal strategy.

In [3] two restricted ways to apply a term rewriting system (TRS) to a tree are considered. When the *one-pass root-started* strategy is followed, rewriting starts from the root and continues stepwise towards the leaves without ever rewriting a part of the current tree produced in a previous rewrite step. *One-pass leaf-started rewriting* is defined similarly, but rewriting begins from the leaves. In the *sentential form inclusion problem* one asks whether all trees which can be obtained with a given TRS from the trees of some regular tree language  $T$  belong to another given regular tree language  $U$ , and in the *normal form inclusion problem* the same question is asked about the normal forms of  $T$ . We show that for a left-linear TRS these problems can be decided for both of our one-pass strategies. In all four cases the decision algorithm involves the construction of a suitable tree recognizer.

In [7] we consider restricted versions of ground tree transducers: total, deterministic, and symmetric subclasses and all other subclasses created by applying any combination of these restrictions. We present the inclusion diagram of the tree transformation classes induced by these restricted ground tree transducers.

We show that the following four classes of term relations are the same: (i) tree transformations induced by symmetric deterministic ground tree transducers, (ii) congruence relations on term algebras induced by reduced ground term rewriting systems, (iii) congruence relations on term algebras induced by convergent ground term rewriting systems, and (iv) finitely generated congruence relations on term algebras.

As a by-product of our results, we obtain a new ground completion algorithm.

Moreover, we show that the following three classes of term relations on term algebras with at least one non-nullary function symbol are also the same: (i) tree transformations induced by total symmetric deterministic ground tree transducers, (ii) congruence relations on term algebras of finite index, (iii) finitely generated congruence relations on term algebras of which the trunk is the whole set of terms.

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Structure theory is a traditional area of automata theory that has been the source of many deep results on this field. Properties of the members of special classes of automata and nondeterministic automata render it possible to

- (1) characterize the subdirect irreducible members of the class considered,
- (2) help to describe the isomorphically and homomorphically complete systems for specific classes of automata under different compositions,
- (3) describe the languages accepted by the members of the classes under consideration,
- (4) study some questions connected with the directable automata.

The results concerning the structural investigations of the authors are summarized below.

Regarding the subdirectly irreducible automata, three special classes, the classes of definite, reverse-definite and generalized definite automata are studied in [3], where the subdirectly irreducible members of these classes are characterized. By using structural properties, originated from semigroup theory, a general description of the subdirectly irreducible members of the class of automata is presented in [1].

As far as the isomorphically complete systems are concerned, in [5] the class of the generalized definite automata is studied and the isomorphically complete systems of generalized definite automata with respect to the  $\alpha_i$ -products are characterized. From this characterization it follows that there is no finite isomorphically complete system for this class with respect to the  $\alpha_0$ -product, while there is a singleton isomorphically complete system for the class under consideration with respect to any  $\alpha_i$ -product with  $i \geq 1$ . Another specific class, the class of the commutative asynchronous automata is studied in [14] where the isomorphically complete systems of the commutative asynchronous automata are described with respect to the Glushkov-type product. More general classes, the class of monotone tree automata, and the class of nondeterministic monotone tree automata are investigated in [6] where the isomorphically complete systems are characterized for these classes with respect to the  $\alpha_0$ -products. From these descriptions it follows that there are singleton isomorphically complete systems for both classes with respect to the  $\alpha_0$ -product. In [4] a structural characterization of three classes of tree automata is presented, namely, the classes of nilpotent, definite, and monotone tree automata are homomorphically represented by means of quasicascade-products of unary nilpotent and unary definite tree automata in the first two cases, and by means of products of simpler tree automata in the third case. The isomorphically complete systems for the class of all automata under different compositions are studied in the papers [10] and [8]. In [10] it is shown that the cube-product is equivalent to the Glushkov's product of automata, and thus, the isomorphically complete systems collapse for this two compositions. In [8] a general notion of composition is studied which is defined by a graph-class. A sufficient condition is presented for the graph-class which provides that there

exists a finite isomorphically complete system for the composition defined by this graph-class.

Different special classes of languages are described in the papers [7, 9, 11, 12, 13]. In [11, 12] the classes of languages consisting of the languages containing the directing words of nondeterministic directable automata are described and compared. A similar investigation can be found in [16] for commutative directable nondeterministic automata. A small particular language class, the class of the languages accepted by commutative asynchronous automata is described in [13]. Another special class, the class of the languages recognizable by monotone automata is characterized in [7]. The tree languages recognized by deterministic root-to-frontier tree automata are characterized in [9].

In [15] the directable commutative asynchronous automata are studied and a tight upper bound is presented for the maximum of the shortest directing words of  $n$ -state directable commutative asynchronous automata. The notion of the directability of automata is extended to nondeterministic automata in [17] where lower and upper bounds of the maximum of the shortest directing words of  $n$ -state directable nondeterministic automata are investigated. This extension yielded further studies (see [11, 12, 16]).

## RELATIONSHIP BETWEEN THIS RESEARCH TOPIC AND THE EDUCATION

Results concerning the structure studies of automata can constitute different PhD courses, furthermore, they provide numerous possibilities for further research work.

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# DEPENDENCE ANALYSIS OF DECLARATIVE PROGRAMS AND THEIR APPLICATIONS IN MACHINE LEARNING PROBLEMS

Tibor Gyimóthy, Gyöngyi Szilágyi and Szilvia Zvada

Attribute Grammars (AG) are generalizations of the concept of Context-Free Grammars. The formalism of AGs has been widely used for the specification and implementation of programming languages. On the other hand there is a close relationship between AGs and another declarative approach, (Constraint) Logic Programming (LP) an emerging software technology with growing number of applications. Data analysis of (Constraint) Logic Programs is useful in finding the connected components of programs, plays an important role for example in debugging, testing and program maintenance.

Efficient techniques for learning attribute grammars can help in finding good definitions of AGs, which usually require a lot of effort.

In this short study we give a summary of results achieved during 1998-2001 in the fields of program analysis (slicing) of (Constraint) Logic Programs, and the learning of semantic functions of Attribute Grammars.

## I. DATA FLOW ANALYSIS OF LOGIC PROGRAMS

Slicing is a program analysis technique originally developed for imperative languages. It can be applied in a number of software engineering tasks, is a natural tool for debugging, is useful in incremental testing, and can help to detect dead code or to find parallelism in programs. Intuitively, a program slice with respect to a specific variable at some program point contains all those parts of the program that may affect the value of the variable (backward slice) or may be affected by the value of the variable (forward slice).

Data flow in logic programs is not explicit, and for this reason the concept of a slice and the slicing techniques of imperative languages are not directly applicable. Moreover, implicit data flow makes the understanding of program behavior rather difficult. Thus program analysis tools explaining data flow to the user are of great practical importance. Papers [1, 2, 3] extends the scope and optimality of previous algorithmic debugging techniques of Prolog programs using slicing techniques. They provide dynamic slicing algorithms augmenting the data flow analysis with control flow dependence to help one locate the connected components of a program and the source of a bug included in the program. A tool has been developed for debugging Prolog programs which also handles the specific programming techniques (cut, if-then, or).

## II. DATA FLOW ANALYSIS OF CONSTRAINT LOGIC PROGRAMS

Constraint Logic programming is a fusion of two declarative paradigms: constraint solving and logic programming. The framework extends classical logic programming by removing the restriction on programming within the Herbrand universe alone; unification is replaced by the more general notion of constraint satisfaction. In [4, 5, 6] we formulate declarative notions of a slice suitable for CLP. (The problem of find-

ing minimal slices may be undecidable in general, since satisfiability may be undecidable.) Our slice definition provides a basis for defining slicing techniques based on variable sharing. The techniques are further extended by using groundness information. We also present a prototype slicing tool for CLP programs written in SICStus Prolog.

## III. LEARNING SEMANTIC FUNCTIONS OF ATTRIBUTE GRAMMARS

In the framework of compilation oriented language implementation, Attribute Grammars (AG) are the most widely applied semantic formalism. Since the definition of an AG and its semantic functions may be complex it is very useful to have an efficient tool for inferring semantic rules of AGs from examples. Based on the correspondence of Attribute Grammars and Logic Programs some of the learning technique developed for Logic Programs (ILP) could be applied to AGs. Introducing an AG based description language in ILP implies the definition of an Attribute Grammar Learner. In [7] we present a parallel method for learning semantic functions of Attribute Grammars based on an ILP approach. The method given is adequate for S-attributed and for L-attributed grammars. The parallelism can help to reduce the large number of user queries posed during the interactive learning process.

In [8, 9] a learning method called LAG is presented, which infers semantic functions for simple classes of AGs by means of examples and background knowledge. LAG generates the training examples on its own via the effective use of background knowledge. The LAG method makes use of the C4.5 decision tree learner during the learning process. This method has been applied to the Part-of-Speech tagging of Hungarian sentences, as well.

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Most of the research done in the interval 1999-2001 is included in my Ph.D. thesis. [5]. Research conducted in artificial speech recognition [9, 8] is covered elsewhere.

## I. HEURISTIC GLOBAL OPTIMIZATION

This line of research falls under the field of evolutionary computing. [5, 6] describe global optimizers that also explore the structure of the local optima of an optimization problem. The first version is called GAS. This algorithm is able to make use of structural information adaptively using a niching technique based on a subpopulation approach simulating the formation of species. This means that the search space is divided to clusters (which we call species), and these cluster-structure is constantly modified according to the results of global search restricted to these clusters. The cluster structure reflects the distribution of local optima.

The bias of the algorithm can be controlled explicitly, as the structure of the space (the distance function of the space) can be implemented arbitrarily, and the automatic parameter setting of some important parameters is possible based on this distance function and other user-given parameters. The output of the algorithm contains the structural information as well as the final solution. The second version is called UEGO.

The difference from GAS is that the optimizer that works inside of a species can be any algorithm. This turns the approach into a general hybrid paradigm that can be applied along with any optimizer if the structure of the local optima is important or if stable solutions must be found. Another difference is that the structure of the algorithm was simplified to make parallel implementation easier. For results of the parallel implementation please consult [11].

## II. EVOLUTIONARY COMPUTATION THEORY

[7] offers a way of visualizing the structure of spaces and also provides a measure of problem difficulty. In this case the visualization is done based on the evaluation function and the algorithm at hand so it is a tool to look at the structure defined by the evaluation function from the viewpoint of the search algorithm. In 2001 Boglárka Tóth and Tamás Vinkó won the 1st prize at the National Competition of Students (OTDK) in this subject area under my supervision.

[4] offers an approach that makes this automatic process possible. A framework is presented in which binary encodings of a problem domain can be learned using ideas from machine learning. The actual learning procedure can be arbitrary. Using such an approach, it may be possible in the future to solve the reverse engineering problem in a similar way evolution solved it for us evolving “devices” such as the eye and human language. The later — besides its other functions — allows us to describe the human domain, i.e. the aspects of the world that are important for us. This makes it possible to plan our actions and to

predict future events efficiently. In abstract domains a similar approach of evolving such representations that allow efficient search, information compression and even communication is becoming computationally feasible.

## III. OTHER GENETIC COMPUTATION

[1] it is suggested that the way of looking at evolution in the field of evolutionary computation is very similar to the so called *adaptational stance*. It is also shown that — beside the similarities — there are significant differences as well, and these are connected to the epistemological crisis of computational science. The point is that since biologists have a natural vocabulary for talking about features of things (wing, eye, hart, etc.) in computer science the situation is radically different. To imagine this try to talk about an elephant referring only to its DNA sequence; well, in evolutionary computation people have no other information. I also propose that the problem is deep and we have no grounds to think that it will ever be solved by humans. Human intelligence was evolved in a specific environment for specific tasks. Though the emergence of language made it possible to reach high levels of abstraction, to really describe such spaces and to make use of their structure the only way might be to do it automatically using artificial intelligence and machine learning where concepts that are completely un-natural for humans can be handled and evolved.

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To create a spatial database for some GIS (Geographic Information System) application, it is a big challenge to recognize all the simple and complex map objects automatically on scanned maps. This research field is termed as *map interpretation*. Related topics are OCR (Optical Character Recognition), *form analysis* and *engineering drawing interpretation*.

## I. APPROACHES

Published map interpretation systems can be classified into three categories:

- Raster-based systems perform all recognition on scanned raster data.
- Vector-based solutions start with a raw vectorization, and all the recognition steps are performed on vector data.
- Hybrid solutions usually perform segmentation to separate text from graphics, and text is recognized with some raster-based OCR technique while graphics is analyzed after vectorization.

In our work we prefer the vector-based approach discussed below.

## II. INTERPRETATION OF CADASTRAL MAPS

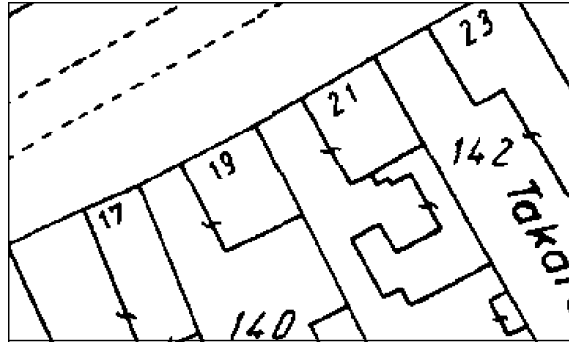
A new data model - called DG (Drawing Graph) - has been developed [5] to support vector-based recognitions. A preliminary version of DG has been used in a signature verification system, an advanced version is applied for interpretation of Hungarian cadastral maps [1]. Our model involves four object types:

- NODE is a point with coordinates (x, y),
- EDGE is a straight line segment between two nodes,
- TEXT is an inscription on the map,
- PAT is an arbitrary set of other objects.

We show that these four types are sufficient to describe all complex structures required in map interpretation. The DG model can be discussed both on logical (abstract) level and on physical (machine data structure) level. Considering the latter one, it can be shown that initial topology generation takes  $\mathcal{O}(N \cdot \log(N))$  time for N vectors, and all update operations take constant time [3]. As a conclusion, DG is a simple, universal and efficient model and in this sense it is prior to other data structures applied in map interpretation.

The DG model has been implemented in the interpretation system MAPINT. Although the system is basically universal, it has special support for Hungarian cadastral maps [1]. Processing starts with an affine coordinate transform followed by raw vectorization, the result of which is converted into DG format. At this stage of processing DG contains only NODE and EDGE objects, while TEXT and PAT objects are generated during recognition. The final result can be exported in DXF format. Interpreted map objects are as follows (Fig. 1):

a,



b,



c,

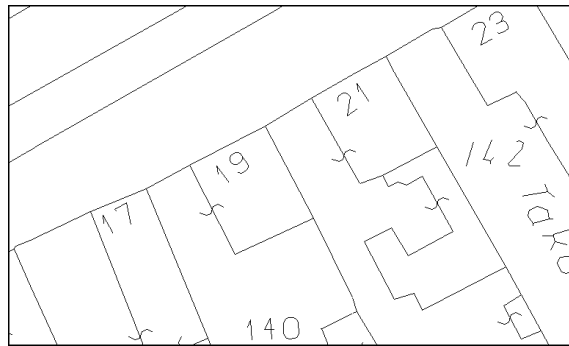


Figure 1. Automatic interpretation of Hungarian cadastral maps: a, scanned raster image, b, raw vectorization, c, result of automatic recognition.

- dashed lines,
- inscriptions (house numbers and parcel numbers),
- connection signs (a special notation of Hungarian maps expressing the relationship between a building and a parcel),
- null-circles (denoting geodetical identified points),
- building and parcel polygons.

Character recognition is performed by a feed-forward back-propagation neural network ensuring

learning abilities for the system. We show that practically all recognition operations can be performed in linear time when grid indexing is used for the DG [3].

MAPINT has been applied in the Phare HU 905.0203 Land Consolidation Project to support the creation of a Hungarian cadastral map database. In this project MAPINT has served for recognizing parcel numbers and connecting them with parcel records stored in an Oracle database [2].

### III. TERRAIN INTERPRETATION

Thin plate spline interpolation is a widely used approach to generate a digital elevation model (DEM) from contour lines and elevation dots. In practice, contour maps are scanned and vectorized, and after resampling in the target grid resolution, interpolation is performed. In [3] we reveal the restricted accuracy of this process, and propose a high resolution processing method (without vectorization) ensuring maximum utilization of information involved in source data.

There are several ways to create a DEM from a scanned contour map. We prefer the thin plate spline interpolation with multigrid relaxation to speedup convergence. This method works well for scattered data points, but it gives incorrect result when applied for contour maps in scanning resolution. To overcome the problem, a *contour thinning reduction method* [4, 6] has been developed and applied in the multigrid process. This method gives correct result even if contour lines and elevation dots are processed at the same time (Fig. 2).

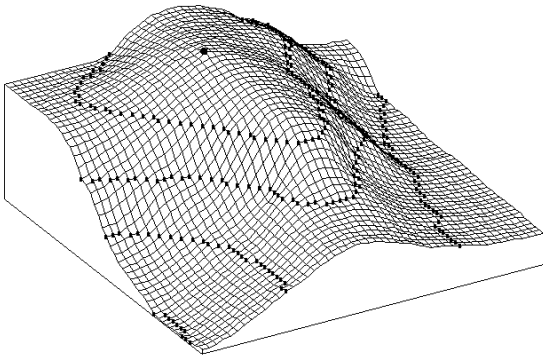


Figure 2. Result of multigrid relaxation with contour thinning reduction.

An earlier version of our technology has been applied in a Hungarian project producing the first DEM covering the whole country with 10 x 10 meter resolution. To overcome the problem of computation time of high resolution processing, a special parallel processor has been applied to speed up convolution. Note that the power of current computers is enough to apply our method without any additional hardware.

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# CURRENT RESEARCH & DEVELOPMENT IN SPEECH RECOGNITION

András Kocsor, László Tóth, and János Csirik

## I. MOTIVATION

When choosing the directions of our speech recognition research, we decided to focus on Hungarian with the hopes that we can address some special issues concerning the processing of our national language, and also that we can make use of our previous experience with NLP for Hungarian. Furthermore, we were looking for a flexible framework that allows experimentation with different preprocessing techniques, feature-space transformation methods and machine learning algorithms. These expectations led us to the stochastic segmental approach which, in a certain sense, can be viewed as an extension of hidden Markov modelling.

## II. THE "OASIS" RECOGNITION SYSTEM

Our recognition system follows the stochastic segmental approach mentioned above and was named "OASIS" [7, 8, 10] (from Our Acoustics-based Speaker-Independent Speech recognizer). The system was designed to be as modular as possible, so we can easily conduct experiments with combining different techniques for the several subtasks of recognition. The first module is the usual frame-based preprocessing phase. After this an additional step, the modelling of phonetic segments was inserted. This allows the combination of different preprocessing techniques and also incorporation of phonetic knowledge. The output of this module are the so-called segmental features, which are used by phoneme classifiers for the classification of the segment. For this task, again, several learning algorithms can be applied. Finally, the segmental classification scores are combined by the matching engine, which performs an utterance-level search in the graph of the possible segmentations. Its goal is to find the best matching transcription from those provided by the language model. The system makes it possible to experiment with many search techniques and strategies.

## III. DISCRIMINATIVE PHONEME CLASSIFICATION

The main advantage of the segmental approach from our point of view is that one can apply many traditional pattern recognition algorithms for the phoneme classification task. In a comparative study we found that discriminative classifiers like Artificial Neural Nets or Support Vector Machines can significantly outperform the usual HMM technique [9].

## IV. NON-LINEAR FEATURE SPACE TRANSFORMATIONS

These classifiers can be made even more efficient with the aid of feature-space transformations. One of our main research interest currently is the application of non-linear transformations for phoneme classification. We wrote several studies investigating the efficiency

of such transformations as Kernel-LDA, Kernel-PCA and Kernel-ICA [11, 12, 13, 14].

## V. RUNNING PROJECTS

A very important issue in statistical speech recognition is the training database. Without a proper corpus the recognition system cannot be trained reliably. Unfortunately, there are no such corpora for Hungarian which would be large enough to train large vocabulary continuous speech recognizers. In the framework of the national grants "IKTA" we are currently working on such a large telephone-based speech corpus in cooperation with the Technical University of Budapest. We have already made a small corpus containing natural numbers, and our recognizer is trained and tested on this database. This corpus will be made available to any research institute in Hungary with the financial aid of governmental funds, within the framework of grant SZT-IS-10. This project also contains the creation of free source-code algorithms that aim to encourage speech recognition research and development in Hungary.

Besides, we have applied for grants in the field of speech impediment therapy (see below).

## VI. SPEECH IMPEDIMENT THERAPY

A very interesting application of automatic speech recognition is the speech therapy of the hearing impaired, as this poses the most serious challenge for phoneme recognition. We study the applicability of our phoneme classification methods to this field [13]. Our cooperative partner in this project is the "Elementary School of Deaf Children".

## VII. SOFTWARE

As regards to softwares, the most important result of our development is the "OASIS" modular system. It was developed in MS Visual C++ and lately was converted into a library. In this modular environment one can very easily conduct experiments with several feature extraction, phoneme classification and word recognition techniques. The number of functions contained in the library is dynamically extending as newer and newer routines are being added.

For the manual segmentation and labeling of our corpora we developed a software tool that we named "Segmentation Assistant". Besides the graphical environment, this software also lends segmentation algorithms to speed up manual segmentation. We intend to develop this program further by incorporating our recognizer into it, which is hoped to suggest even better segmentations.

Furthermore, we recently developed a software tool called "Learning Assistant", which gives a graphical interface to study the result of combining various feature space transformation and learning methods. The software already includes several linear and non-linear transformation algorithms and learning techniques, and is continuously being extended.

## VIII. HUMAN RESOURCES

Currently we have 3 researchers, 3 Phd students and 2 programmers who work on the speech project. We enlist their names and expertise below. Besides them there are about 6-7 graduate students who take part in the software development.

Prof. János Csirik, head of the group: project management, pattern matching algorithms

Dr. Tibor Gyimóthy, deputy head: project management, machine learning algorithms

András Kocsor, research assistant: feature transformations, pattern classification

László Tóth, research assistant: signal processing, feature extraction

Kornél Kovács, PhD student: feature transformations, machine learning

Gábor Gosztolya, research assistant: machine learning

László Felföldi, PhD student: development of the modular environment of "OASIS"

Dénes Paczolay, PhD student: machine learning in speech impediment therapy

András Bíró, programmer: development of the "Segmentation Assistant"

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Thinning is an iterative object reduction technique that provides an approximation to the skeleton in a topology preserving way. It means that the result of the thinning process is topologically equivalent to the original object. Our attention is focused on developing 3D thinning algorithm that are to be used in medical applications. This report describes the skeleton, the major skeletonization techniques in the case of discrete objects, the thinning methodologies, and our results.

## I. SKELETON AS A SHAPE FEATURE

Shape is a fundamental concept in computer vision. It can be regarded as the basis for high-level image processing stages concentrating on scene analysis and interpretation. There are basically two different approaches for describing the shape of an object:

- using the boundary that surrounds it and
- using the occupied region.

Boundary-based techniques are widely used but there are some deficiencies which limit their usefulness in practical applications especially in 3D. Therefore, the importance of the region-based shape features show upward tendency. The local object symmetries represented by the skeleton certainly cannot replace boundary-based shape descriptors, but complement and support them.

The skeleton is a region-based shape feature that has been proposed by Blum as the result of the Medial Axis Transform. A very illustrative definition of the skeleton is given using the prairie-fire analogy: the object boundary is set on fire and the skeleton is formed by the loci where the fire fronts meet and quench each others. The formal definition of the skeleton has been stated by Calabi: the skeleton of an object is the locus of the centers of all the maximal inscribed hyperspheres. The continuous skeleton of a solid 3D box is illustrated in Fig. 1.

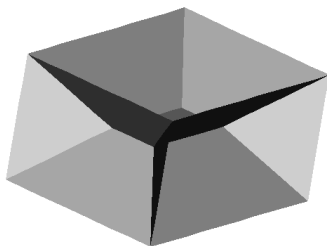


Figure 1. Example of 3D skeleton. The original object was a solid box.

## II. SKELETONIZATION TECHNIQUES IN DISCRETE SPACES

During the last two decades skeletonization in the digital image raster has been an important research field. There are two major requirements to be complied with. The first one is geometrical. It means that the “skeleton” must be in the “middle” of the object and invariant under geometrical transformations. The

second one is topological requiring that the “skeleton” must be topologically equivalent to the original object.

There are three major discrete skeletonization methods:

- based on distance transformation,
- thinning, and
- based on Voronoi-diagram.

The first method is to find the maximal inscribed hyperspheres. It requires the following 3-step process:

1. The original binary picture is converted into another one consisting feature and nonfeature elements. The feature elements belong to the boundary of the discrete object.
2. The distance map is generated where each element has a value that approximates the distance to the nearest feature element.
3. The detection of ridges (local extremas) as the centers of maximal inscribed hyperspheres.

Unfortunately, the result of the distance transformation depends on the selected distance and the ridge extraction is a rather difficult task. The distance map based method fulfills the geometrical requirement if a good approximation to the Euclidean distance is applied, but the topological correctness is not guaranteed.

The thinning process is to simulate the fire-front propagation: a layer by layer erosion is executed until the “skeleton” is left. The iterative process is shown in Fig. 2. The topological aspect is taken care by thinning. On the other hand the geometrical requirement is not satisfied.

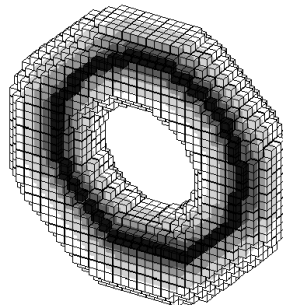


Figure 2. Phases of a thinning process. Results of the layer by layer deletion steps are denoted by different grey-levels. The darkest points belong to the “skeleton”.

The Voronoi diagram of a discrete set of points (called generating points) is the partition of the given space into cells so that each cell contains exactly one generating point and the locus of all points which are nearer to this generating point than to other generating points. It is shown that the skeleton of an object which is described by a set of boundary points can be approximated by a subgraph of the Voronoi diagram of that generating points.

Both requirements can be fulfilled by the skeletonization based on Voronoi diagrams but it is regarded as an expensive process, especially for large and complex objects.

We prefer thinning, since it:

- preserves topology,
- makes easy implementation possible (as a sequence of local Boolean operations),
- takes the least computational costs, and
- can be executed in parallel.

### III. THINNING METHODOLOGIES

A 3D binary picture is a mapping that assigns value of 0 or 1 to each point with integer coordinates in the 3D digital space denoted by  $\mathbb{Z}^3$ . Points having the value of 1 are called black points, while 0's are called white ones. Black points form objects of the picture. White points form the background and the cavities of the picture. Both the input and the output of a picture operation are pictures. An operation is reduction if it can delete some black points (i.e., changes them to white) but white points remain the same. There is a fairly general agreement that a reduction operation is *not* topology preserving if any object in the input picture is split (into two or more ones) or completely deleted, if any cavity in the input picture is merged with the background or another hole, or if a cavity is created where there was none in the input picture. There is an additional concept called hole in 3D pictures. A hole (that doughnuts have) is formed by 0's, but it is not a cavity. Topology preservation implies that eliminating or creating any hole is not allowed.

Thinning must be a topology-preserving reduction. Existing 3D thinning algorithms can be classified from several points of view. One of them is the classification on the produced skeletons: some of the developed algorithms result in medial surfaces and others can produce medial lines (see Fig. 3).

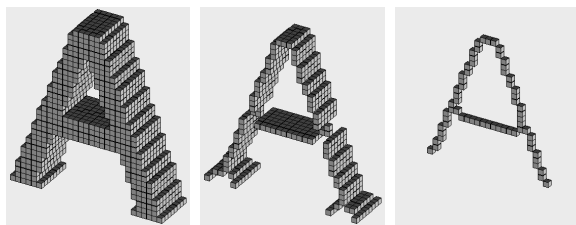


Figure 3. A 3D synthetic picture containing a character “A” (left); its medial surface (centre); its medial lines (right).

Three types of parallel thinning methodologies have been proposed.

The first type examine the  $3 \times 3 \times 3$  neighbourhood of each border point (i.e., black point that has at least one white point in its neighbourhood). The iteration steps are divided into a number of subiterations. Only border points having the prescribed  $3 \times 3 \times 3$  neighbourhood can be deleted during a subiteration. It means that each prescribed neighbourhood gives a deletion condition. Prescribed neighbourhoods can be usually associated to a direction (e.g., up, down) depending on the position of the border points to be deleted. These algorithms use directional or border sequential strategy. Each subiteration is executed in parallel (i.e., all black points satisfying the deletion condition of the actual subiteration are simultaneously deleted). Most of existing parallel thinning algorithms are border sequential. Generally 6 subiterations are used.

A directional algorithm consisting of  $k$  subiterations follows the following process:

```

repeat
  for  $i = 1$  to  $k$  do
    simultaneous deletion of the black points
    that satisfy the condition assigned to the
     $i$ -th direction
until no points are deleted

```

We have developed three different border-sequential 3D thinning algorithms. They use 3, 6, 8, and 12 subiterations, respectively [1, 2, 3, 4]. Fig. 4 presents the results of three curve-thinning algorithms for a synthetic object and Fig. 5 is to show the medial surfaces produced by our 3-subiteration algorithm.

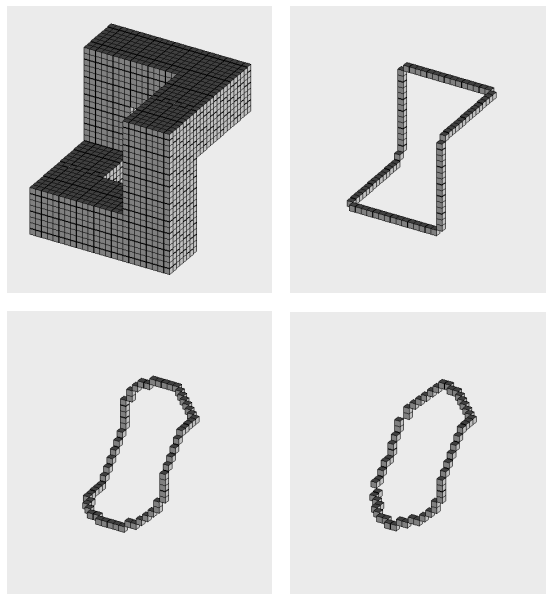


Figure 4. Thinning of a 3D synthetic object of size  $24 \times 24 \times 24$  (upper left). The medial lines produced by the 6-subiteration algorithm (upper right), the 8-subiteration algorithm (bottom left), and the 12-subiteration algorithm (bottom right).

The second type of algorithms does not need subiterations but — in order to preserve topology — it investigates larger ( $5 \times 5 \times 5$ ) neighbourhood. This approach can be sketched by the following program:

```

repeat
  simultaneous deletion of the black points
  that satisfy the global condition
until no points are deleted

```

The third approach is the subfield sequential method. The set of points  $\mathbb{Z}^3$  is divided into more disjoint subsets which are alternatively activated. At a given iteration step, only black points of the active subfield are designated to be deleted.

A subfield based algorithm consisting of  $k$  subfields can be described as follows:

```

repeat
  for  $i = 1$  to  $k$  do
    simultaneous deletion of the black points
    in the  $i$ -th subfield that satisfy the global
    condition assigned to each subfield
until no points are deleted

```

We have proposed a new class of thinning methods called hybrid algorithms. It applies both directional and subfield strategies. A hybrid algorithm containing  $k_1$  directional subiterations and  $k_2$  subfields is described by the following program:



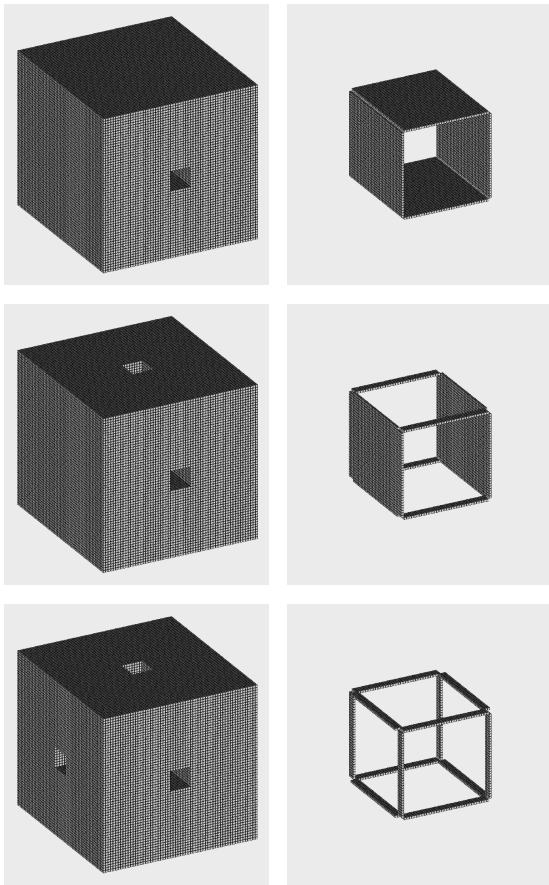


Figure 5. Three 3D synthetic object (left) and their medial surface produced by our 3-subiteration algorithm (right).

```

repeat
  remark the  $k_1$  directional-type subiterations
  for  $i = 1$  to  $k_1$  do
    simultaneous deletion of the black points
    that satisfy the condition assigned to the
     $i$ -th direction
  remark the  $k_2$  subfield-type subiterations
  for  $j = 1$  to  $k_2$  do
    simultaneous deletion of the black points
    in the  $j$ -th subfield that satisfy the global
    condition assigned to each subfield
until no points are deleted

```

Our 3D hybrid thinning algorithms uses  $k_1 = 8$  directional-type subiterations and  $k_2 = 2$  subfield-type subiterations [5].

#### IV. APPLICATIONS

Thinning provides “skeleton-like” shape feature that are extracted from binary image data. It is a common preprocessing operation in raster-to-vector conversion or in pattern recognition. Its goal is to reduce the volume of elongated objects. Some important applications have been appeared in medical image processing, too [6, 7, 8, 9].

Fig. 6 presents the central paths of a segmented part of an infrarenal aorta, a segmented upper respiratory tract, and a segmented cadaveric phantom. Fig. 7 presents the central paths of a segmented pulmonary tree.

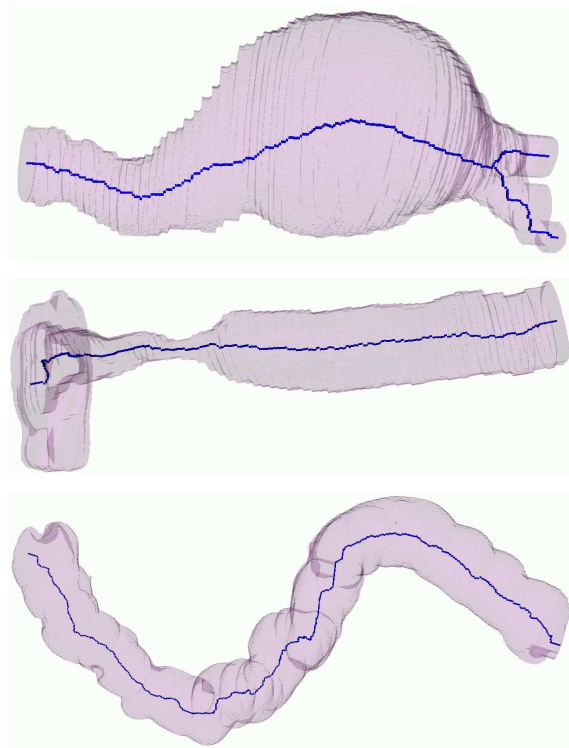


Figure 6. Central paths for a segmented part of an infrarenal aorta (top), a segmented upper respiratory tract (middle), and a segmented cadaveric phantom (bottom) produced by our curve-thinning algorithm. In order to get longer central paths, the endpoints were automatically identified before the thinning and those points were regarded as “anchors” during the thinning process (i.e., their deletion were prohibited). In that way, the topologically correct thinning algorithm was urged to connect them.

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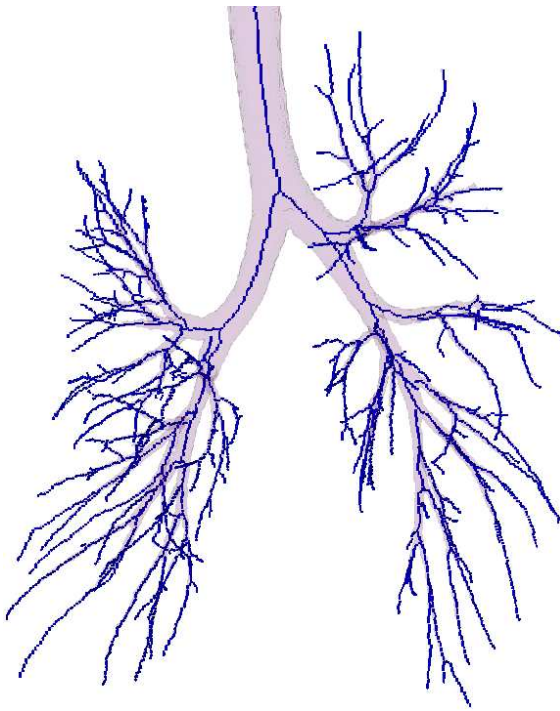


Figure 7. Central path of a pulmonary phantom produced by our curve-thinning algorithm.

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Image registration is a fundamental task in digital image processing used to match two independently acquired images. These images are taken at different times, from different viewpoints or even from different imaging devices. To register images, the geometrical relationship between them is to be determined. Image registration is an important area of medical image processing. Matching all the geometric data available for a patient provides better diagnostic capability, better understanding of data, and improves surgical and therapy planning and evaluation. This report describes the registration problem, the phases of the process, the proposed methods, and our results.

## I. REGISTRATION PROBLEM

There is an increasing number of medical applications that require accurate aligning of one medical image with another. The geometrical transformation is to be found that maps a *reslice image data set* in precise spatial correspondence with a *base image data set*. This process of alignment is known as *registration*, although other words, such as *co-registration*, *matching* and *fusion*, also are used.

In the course of many medical procedures different types of images are acquired. Each method is referred to as a *modality*. Medical imaging technologies are based on physical properties, including x-rays, magnetic properties, distribution of radiopharmaceuticals, ultrasound, and various others. Each modality has its own assets and reveals different properties of the human body. The common modalities providing 3D image data are Computerised Tomography (CT), Magnetic Resonance Imaging (MRI), Single Photon Emission Computed Tomography (SPECT) and Positron Emission Tomography (PET). The existing imaging modalities can be divided into two categories: *anatomical* and *functional*. Anatomical (or morphologic) images (CT and MRI) offer good visualization of tissues. CT is best for 3D visualization of bone, MR for soft tissues and nerves. Functional images (SPECT and PET) have relatively poor spatial resolution but these modalities (referred to as modalities of *nuclear medicine*) provide information concerned with metabolism which is not available with other techniques.

One may distinguish between *multimodality* and *unimodality registration*. Multimodality registration is used to match 2D and 3D anatomical images with each other and/or with 3D functional images. Aligning the complementary information content of the images from different modalities means a great benefit during diagnosis and treatment planning. An eminent example of multimodality registration is in the treatment of cancer by radiotherapy. An MRI data set is capable of showing the tumour while a CT scan shows the skeletal structure. The radiotherapy dose can be calculated with greater accuracy using these combined images. Besides multimodality registration, important applications exist in unimodality registration. It is useful to combine images in time-series or even to align images with different protocol settings (e.g., with and without contrast injection). It is

commonly used to show changes between images over a period of time. The progress of a disease can be tracked or the efficiency of the applied treatments can be followed.

## II. REGISTRATION TECHNIQUES

Medical image registration techniques can be classified from several points of view. One of them is the classification on matching properties (nature of registration basis): *intrinsic* (patient related) and *extrinsic* (artificial marker related). Extrinsic registration methods rely on artificially attached objects which are designed to be well detectable in all of the applied modalities. Extrinsic information is supplied by, for example, stereotactic frames, other head and dental fixation devices, or skin markers. Intrinsic registration uses patient generated image content only. It may be based on anatomical landmark points, objects (e.g., skull, brain, ventricles), or geometrical image features (e.g., local curvature extrema, corners).

Three types of registration techniques can be distinguished:

- *manual*, where the two images to be registered are displayed (e.g., on different colour planes) and the reslice image is moved manually until a relevant matching is achieved;
- *interactive*, where features are given by the user in both images and then the positions of these features are used to calculate the required transformation;
- *automated*, where a program is to be run making the registration without any intervention.

Both manual and interactive techniques require anatomical knowledge while an automated system can be run by an unprofessional user.

Registration techniques depend on image content; thus no general methodology has been proposed. A variety of registration methodologies have been developed. Registration methods can be viewed as different combinations of choices for the following four components:

- *search space*  
what kind of transform we have to consider, i.e., what is the class of transformations that is capable of aligning the images;
- *feature space*  
what to match, i.e., what are the features to be used in matching;
- *similarity measure*  
how to evaluate the match;
- *search strategy*  
how to match, i.e., the process for achieving a consistent match.

The registration process consists of the following four steps:

- *Feature extraction*  
Feature data sets  $F_1$  and  $F_2$  are to be extracted from the base image  $I_1$  and the reslice image  $I_2$ , respectively;

- *Finding a consistent transformation*  
Transformation  $\mathcal{T}$  is to be determined, where  $S(F_1, \mathcal{T}(F_2))$  is maximized or minimized in the sense of the selected similarity measure  $S$ ;
- *Applying the found geometrical transformation*  
Resampled image  $\mathcal{T}(I_2)$  is to be calculated;
- *Using registered data sets*  
Registered image data sets  $I_1$  and  $\mathcal{T}(I_2)$  may be used in various ways (e.g.,  $I_1$  is displayed overlaid on top of  $\mathcal{T}(I_2)$ ).

### III. REGISTRATION BASED ON FUZZY OBJECTS

We have developed an automated registration method that is used effectively in Multiple Sclerosis projects of Medical Image Processing Group, Department of Radiology, University of Pennsylvania, Philadelphia. It works within the 3DVIEWNIX software system.

The most important component of a registration method is the selection of the feature space. There are two aims of feature extraction. On the one hand, it removes extraneous and irrelevant information from the raw data sets. On the other hand, it generally reduces the amount of data to be evaluated.

Our registration method is based on fuzzy objects used as features. Approaches to object information extraction from images should attempt to use the fact that images are by nature fuzzy. The notion of “hanging togetherness” of image elements specified by their fuzzy connectedness is to be considered in image segmentation research. Udupa and Samarasekera have developed a theory of fuzzy objects for  $n$ -dimensional digital spaces based on a notion of fuzzy connectedness of image elements. The theory and the associated algorithms are described in.

Our approach is described by giving the four components:

- *search space*: rigid body transformation parameterized by 6 degrees of freedom;
- *feature space*: fuzzy objects (or raw image data);
- *similarity measure*: correlation;
- *search strategy*: multi-resolution pyramid.

The major advantage of the method compared to direct correlation of image intensities is that it computes new images representing “objectness” in which regions to be matched are likely to have similar intensities, although this may not be true for the original images.

### IV. REGISTRATION BASED ON POINT LANDMARKS

We have developed a method based on interactively identified anatomical landmark points for multimodality and unimodality registration [1].

The applied method can be described by the following components:

- *search space*:  $4 \times 4$  general transformation matrix capable of giving arbitrary affine transformations;
- *feature space*: anatomical landmark points;
- *similarity measure*: sum of distances;
- *search strategy*: least square minimising of the distance function.

Our method is capable of finding the optimal global affine transformation that matches two sets of points in arbitrary dimensions. We have managed to give

and prove a sufficient existence condition for a unique transformation [3, 4].

We have implemented the method by developing the SIR (Simple Image Registration) software system. It is capable of solving 2D/2D and 3D/3D registration, reslicing images according to the given geometrical transforms and displaying of combined images. The identification of the landmark points is to be done interactively on a computer screen. It has clear advantages in robustness and computation time. The disadvantage is that the identification of the landmark points requires user expertise.

Two additional methods are also implemented for searching rigid body transform and nonlinear warping based on thin-plate spline interpolation.

The SIR software system running on IBM PC-s under Windows operating system is capable of displaying 2D and 3D medical images, resizing and rescaling them, identifying landmark points, applying the found transformation, and image fusion (see Figs. 1-2.).

It can handle various data formats including MicroSEGAMS, Analyze, and DICOM. 3D image data sets can be displayed as a sequence of *transversal*, *sagittal*, and *coronal* sections to be parallel to the  $xy$ -, the  $yz$ -, and the  $xz$ -plane, respectively. SIR makes 3D rendered displaying possible, too. Nearest neighbour, linear and trilinear interpolation techniques can be selected in reslicing. True colour overlay and alternate pixel overlay methods can be used for image fusion. The developed system is not only capable of matching but it can be used for displaying and analyzing the results of other registration methods as well.

We have investigated the registration error for point-based registration [2]. Numerical simulations were performed for four kinds of registration methods assuming different search spaces. We showed that the (target) registration error depends not only on the number of homologous points, and on the accuracy of point localization, but on the volume spanned by the fiducials as well.

### V. REGISTRATION BASED ON MUTUAL INFORMATION

Our fully automated registration method utilizes mutual information as similarity measure. The CT or PET image is resampled to the voxel size of the MR image. The intensities are scaled to [0,127] for both images, then a multiresolution pyramid is built from the 2D slices. A variant of Powell's multidimensional minimization technique is used to find the transformation. At lower resolution nearest neighbour, at the final level bilinear interpolation between slices is used.

Our method has been validated within the Retrospective Registration Evaluation Project (National Institutes of Health, Project Number 1 R01 NS33926-01, Principal Investigator, J.M. Fitzpatrick, Vanderbilt University, Nashville, TN). Results can be found at

[www.vuse.vanderbilt.edu/~jayw/results.html](http://www.vuse.vanderbilt.edu/~jayw/results.html)

More details are to be read at [www.inf.u-szeged.hu/~tanacs/regist/regidx.html](http://www.inf.u-szeged.hu/~tanacs/regist/regidx.html)

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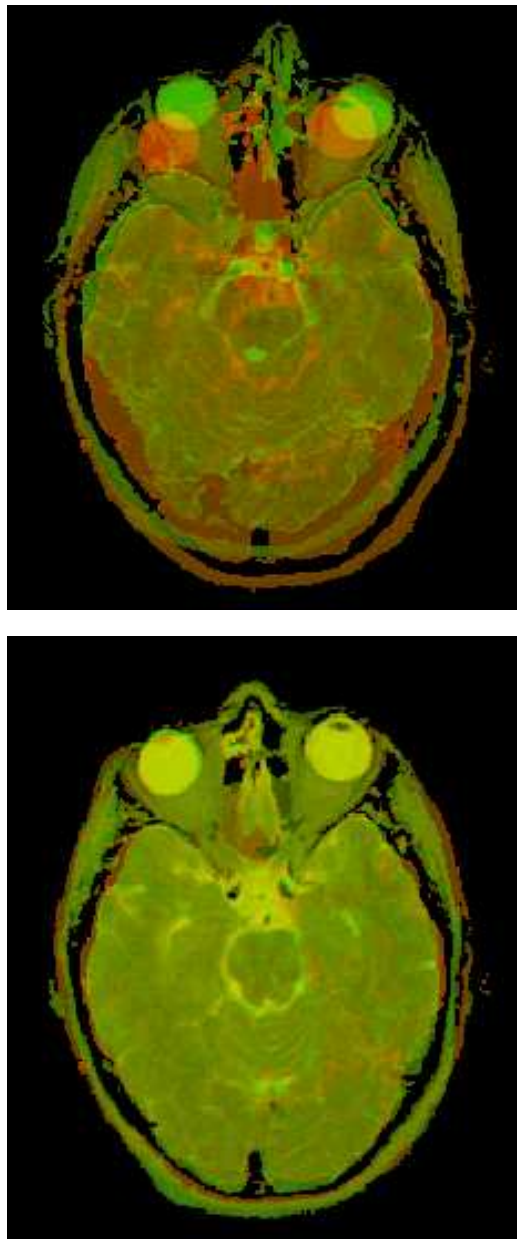


Figure 1. Example of 3D image fusion by SIR system. Overlaid MR transversal 2D slices before registration (top) and after registration (bottom).

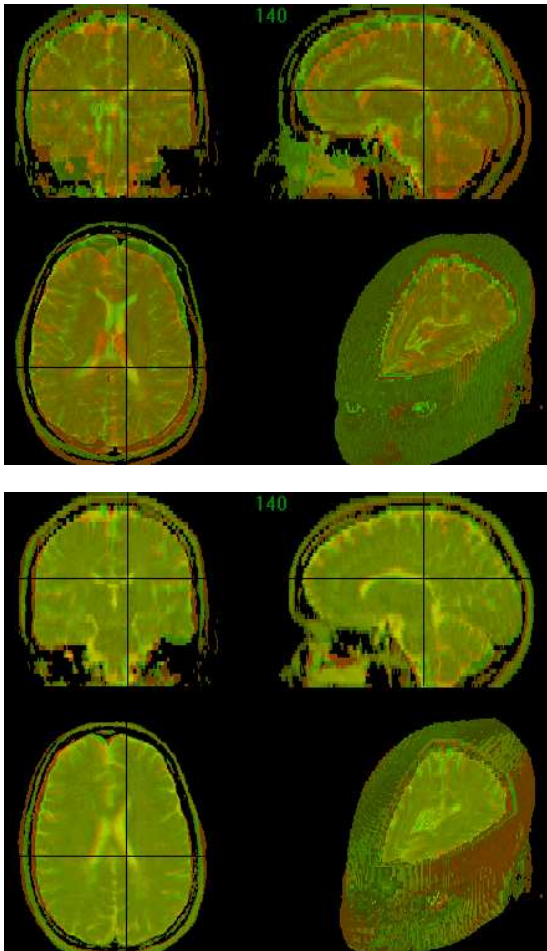


Figure 2. Example of 3D image fusion by SIR system. Overlaid MR coronal, sagittal, transversal 2D slices and a rendered 3D view before registration (top) and after registration (bottom).

**I. TERM REWRITE SYSTEMS EFFECTIVELY PRESERVING RECOGNIZABILITY**

Tree automata and recognizable tree languages proved to be an efficient tool in the theory of term rewrite systems. Let  $\Sigma$  be a ranked alphabet, let  $R$  be a term rewrite system over  $\Sigma$ , and let  $L$  be a tree language over  $\Sigma$ . Then  $R_{\Sigma}^*(L)$  is the set of descendants of trees in  $L$ . When  $\Sigma$  is apparent from the context, we simply write  $R^*(L)$  rather than  $R_{\Sigma}^*(L)$ . A term rewrite system  $R$  over  $\Sigma$  preserves  $\Sigma$ -recognizability, if for each recognizable tree language  $L$  over  $\Sigma$ ,  $R_{\Sigma}^*(L)$  is recognizable. The signature  $sign(R)$  of a term rewrite system  $R$  is the ranked alphabet consisting of all symbols appearing in the rules of  $R$ . Gilleron showed that for a term rewrite system  $R$  it is not decidable if  $R$  preserves  $sign(R)$ -recognizability. Otto showed that it is not decidable for a term rewrite system  $R$  whether  $R$  preserves recognizability. A term rewrite system  $R$  over  $\Delta$  preserves recognizability, if for each ranked alphabet  $\Sigma$  with  $sign(R) \subseteq \Sigma$ ,  $R$  preserves  $\Sigma$ -recognizability. We [3] showed that there is a ranked alphabet  $\Sigma$  and there is a linear term rewrite system  $R$  over  $\Sigma$  such that  $R$  preserves  $\Sigma$ -recognizability but does not preserve recognizability. Let  $R$  be a term rewrite system over  $sign(R)$ , and let  $\Sigma = \{f, \#\} \cup sign(R)$ , where  $f \in \Sigma_2 - sign(R)$  and  $\# \in \Sigma_0 - sign(R)$ . We showed that  $R$  preserves  $\Sigma$ -recognizability if and only if  $R$  preserves recognizability.

Let  $R$  be a term rewrite system over a ranked alphabet  $\Sigma$ . We say that  $R$  effectively preserves  $\Sigma$ -recognizability if for a given tree automaton  $\mathcal{B}$  over  $\Sigma$ , we can effectively construct a tree automaton  $\mathcal{C}$  over  $\Sigma$  such that  $L(\mathcal{C}) = R_{\Sigma}^*(L(\mathcal{B}))$ . Let  $R$  be a term rewrite system over a ranked alphabet  $\Delta$ . We say that  $R$  effectively preserves recognizability if for a given ranked alphabet  $\Sigma$  with  $sign(R) \subseteq \Sigma$  and a given tree automaton  $\mathcal{B}$  over  $\Sigma$ , we can effectively construct a tree automaton  $\mathcal{C}$  over  $\Sigma$  such that  $L(\mathcal{C}) = R_{\Sigma}^*(L(\mathcal{B}))$ . Let  $R$  be a term rewrite system over  $sign(R)$ , and let  $\Sigma = \{f, \#\} \cup sign(R)$ , where  $f \in \Sigma_2 - sign(R)$  and  $\# \in \Sigma_0 - sign(R)$ . We [3] showed that  $R$  effectively preserves  $\Sigma$ -recognizability if and only if  $R$  effectively preserves recognizability.

In spite of the undecidability results of Gilleron and Otto, we know several term rewrite systems which preserve recognizability. Brainerd showed that ground term rewrite systems over any ranked alphabet  $\Sigma$  effectively preserve  $\Sigma$ -recognizability. Gallier and Book introduced the notion of a monadic term rewrite system, and Salomaa showed that linear monadic term rewrite systems over any ranked alphabet  $\Sigma$  effectively preserve  $\Sigma$ -recognizability. A term rewrite system is monadic if each left-hand side is of depth at least 1 and each right-hand side is of depth at most 1. Coquidé et al. defined the concept of a semi-monadic term rewrite system generalizing the notion of a monadic rewrite system and the notion of a ground term rewrite system. A term rewrite system  $R$  over  $\Sigma$  is semi-monadic if, for every rule  $l \rightarrow r$  in  $R$ ,  $depth(l) \geq 1$  and either  $depth(r) = 0$  or  $r = f(y_1, \dots, y_k)$ , where  $f \in \Sigma_k$ ,

$k \geq 1$ , and for each  $i \in \{1, \dots, k\}$ , either  $y_i$  is a variable (i.e.,  $y_i \in X$ ) or  $y_i$  is a ground term (i.e.,  $y_i \in T_{\Sigma}$ ). It is immediate that each monadic term rewrite system is semi-monadic as well. Coquidé et al. showed that linear semi-monadic term rewrite systems over any ranked alphabet  $\Sigma$  effectively preserve  $\Sigma$ -recognizability. We [3] generalized even further the concept of a semi-monadic term rewrite system introducing the concept of a generalized semi-monadic term rewrite system (gsm term rewrite system for short). A term rewrite system  $R$  is gsm if there is no rule  $l \rightarrow r$  in  $R$  with  $l \in X$  and the following holds. For any rules  $l_1 \rightarrow r_1$  and  $l_2 \rightarrow r_2$  in  $R$ , for any occurrences  $\alpha \in O(r_1)$  and  $\beta \in O(l_2)$ , and for any supertree  $l_3 \in T_{\Sigma}(X)$  of  $l_2/\beta$  with  $var(l_3) \cap var(l_1) = \emptyset$ , if

- (i)  $\alpha = \lambda$  or  $\beta = \lambda$ ,
- (ii)  $r_1/\alpha$  and  $l_3$  are unifiable, and
- (iii)  $\sigma$  is a most general unifier of  $r_1/\alpha$  and  $l_3$ ,

then

- (a)  $l_2/\beta \in X$  or
- (b) for each  $\gamma \in O(l_3)$ , if  $l_2/\beta\gamma \in X$ , then  $\sigma(l_3/\gamma) \in X \cup T_{\Sigma}$ .

We [3] showed that a linear gsm (lgs) term rewrite system  $R$  over  $\Delta$  effectively preserves recognizability in the following way. Let  $L$  be a recognizable tree language over  $\Sigma$  with  $\Delta \subseteq \Sigma$ , and let  $\mathcal{B} = (\Sigma, B, R_{\mathcal{B}}, B')$  be a tree automaton recognizing  $L$ . Similarly to the constructions of Salomaa and Coquidé et al., we constructed a sequence of bottom-up tree automata  $\mathcal{C}_i = (\Sigma, C, R_i, B')$ ,  $i \geq 0$  having the same ranked alphabet, state set, and final state set. The rule set  $R_0$  contains  $R_{\mathcal{B}}$ . Moreover,  $R_0$  contains rules which enable  $R_0$  to recognize the right-hand sides of rules in  $R$ . For each  $i \geq 0$ ,  $R_{i+1}$  contains  $R_i$ , and for each rule  $l \rightarrow r$  in  $R$ ,  $\mathcal{C}_{i+1}$  simulates, on the right-hand side  $r$ , the computation of  $\mathcal{C}_i$  on the left-hand side  $l$ . There is a least integer  $M \geq 0$  such that  $R_M = R_{M+1}$ . Hence  $\mathcal{C}_M = \mathcal{C}_{M+1}$ . We [3] showed that  $L(\mathcal{C}_M) = R^*(L)$ .

Brainerd, Kozen, and Fülöp and Vágvölgyi showed that a tree language  $L$  is recognizable if and only if there exists a ground term rewrite system  $R$  such that  $L$  is the union of finitely many  $\leftrightarrow_R^*$ -classes. We [3] showed that a tree language  $L$  is recognizable if and only if there exists a term rewrite system  $R$  such that  $R \cup R^{-1}$  is an lgs term rewrite system and that  $L$  is the union of finitely many  $\leftrightarrow_R^*$ -classes.

We [3] showed the following decidability results.

(1) Let  $R_1, R_2$  be term rewrite systems. Let  $R_1$  effectively preserve recognizability. Then it is decidable if  $\rightarrow_{R_2}^* \subseteq \rightarrow_{R_1}^*$ .

(2) Let  $R_1$  and  $R_2$  be term rewrite systems effectively preserving recognizability. Then it is decidable which one of the following four mutually excluding conditions hold.

- (i)  $\rightarrow_{R_1}^* \subset \rightarrow_{R_2}^*$ ,
- (ii)  $\rightarrow_{R_2}^* \subset \rightarrow_{R_1}^*$ ,
- (iii)  $\rightarrow_{R_1}^* = \rightarrow_{R_2}^*$ ,
- (iv)  $\rightarrow_{R_1}^* \bowtie \rightarrow_{R_2}^*$ ,

where “ $\bowtie$ ” stands for the incomparability relationship.

(3) For an lgs term rewrite system  $R$ , it is decidable whether  $R$  is left-to-right minimal. (A term

rewrite system  $R$  is left-to-right minimal if for each rule  $l \rightarrow r$  in  $R$ ,  $\rightarrow_{R-\{l \rightarrow r\}}^* \subset \rightarrow_R^*$ .

(4) Let  $R_1$  and  $R_2$  be term rewrite systems such that  $R_1 \cup R_1^{-1}$  and  $R_2 \cup R_2^{-1}$  are term rewrite systems and effectively preserve recognizability. Then it is decidable which one of the following four mutually excluding conditions holds.

- (i)  $\leftrightarrow_{R_1}^* \subset \leftrightarrow_{R_2}^*$ ,
- (ii)  $\leftrightarrow_{R_2}^* \subset \leftrightarrow_{R_1}^*$ ,
- (iii)  $\leftrightarrow_{R_1}^* = \leftrightarrow_{R_2}^*$ ,
- (iv)  $\leftrightarrow_{R_1}^* \not\leftrightarrow \leftrightarrow_{R_2}^*$ .

(5) Let  $R$  be a term rewrite system such that  $R \cup R^{-1}$  is an lgsms term rewrite system. Then it is decidable whether  $R$  is two-way minimal. (A term rewrite system  $R$  is two-way minimal if for each rule  $l \rightarrow r$  in  $R$ ,  $\leftrightarrow_{R-\{l \rightarrow r\}}^* \subset \leftrightarrow_R^*$ .)

(6) Let  $R_1, R_2$  be term rewrite systems over a ranked alphabet  $\Sigma$ . Let  $R_1$  effectively preserve recognizability. Let  $g \in \Sigma - \Sigma_0$  be such that  $g$  does not occur on the left-hand side of any rule in  $R_1$ , and let  $\# \in \Sigma_0$  be irreducible for  $R_1$ . Then it is decidable if  $\rightarrow_{R_2}^* \cap (T_\Sigma \times T_\Sigma) \subset \rightarrow_{R_1}^* \cap (T_\Sigma \times T_\Sigma)$ .

(7) Let  $R_1$  and  $R_2$  be term rewrite systems over  $\Sigma$  effectively preserving recognizability. Moreover, let  $g_1, g_2 \in \Sigma - \Sigma_0$  be such that for each  $i \in \{1, 2\}$ ,  $g_i$  does not occur on the left-hand side of any rule in  $R_i$ . Let  $\#_1, \#_2 \in \Sigma_0$  be such that for each  $i \in \{1, 2\}$ ,  $\#_i$  is irreducible for  $R_i$ . Then it is decidable which one of the following four mutually excluding conditions hold.

- (i)  $\rightarrow_{R_1}^* \cap (T_\Sigma \times T_\Sigma) \subset \rightarrow_{R_2}^* \cap (T_\Sigma \times T_\Sigma)$ ,
- (ii)  $\rightarrow_{R_2}^* \cap (T_\Sigma \times T_\Sigma) \subset \rightarrow_{R_1}^* \cap (T_\Sigma \times T_\Sigma)$ ,
- (iii)  $\rightarrow_{R_1}^* \cap (T_\Sigma \times T_\Sigma) = \rightarrow_{R_2}^* \cap (T_\Sigma \times T_\Sigma)$ ,
- (iv)  $\rightarrow_{R_1}^* \cap (T_\Sigma \times T_\Sigma) \not\leftrightarrow \rightarrow_{R_2}^* \cap (T_\Sigma \times T_\Sigma)$ .

(8) Let  $R$  be an lgsms term rewrite system over  $\Sigma$ . Moreover, let  $g \in \Sigma - \Sigma_0$  be such that  $g$  does not occur on the left-hand side of any rule in  $R$ , and let  $\# \in \Sigma_0$  be irreducible for  $R$ . Then it is decidable whether  $R$  is left-to-right ground minimal. (A term rewrite system  $R$  over  $\Sigma$  is left-to-right ground minimal if for each rule  $l \rightarrow r$  in  $R$ ,  $\rightarrow_{R-\{l \rightarrow r\}}^* \cap (T_\Sigma \times T_\Sigma) \subset \rightarrow_R^* \cap (T_\Sigma \times T_\Sigma)$ .)

(9) Let  $R_1$  and  $R_2$  be term rewrite systems over  $\Sigma$  such that  $R_1 \cup R_1^{-1}$  and  $R_2 \cup R_2^{-1}$  are term rewrite systems and effectively preserve recognizability. Moreover, let  $g_1, g_2 \in \Sigma - \Sigma_0$  be such that for each  $i \in \{1, 2\}$ ,  $g_i$  does not occur in  $R_i$ . Let  $\#_1, \#_2 \in \Sigma_0$  be such that for each  $i \in \{1, 2\}$ ,  $\#_i$  is irreducible for  $R_i \cup R_i^{-1}$ . Then it is decidable which one of the following four mutually excluding conditions hold.

- (i)  $\leftrightarrow_{R_1}^* \cap (T_\Sigma \times T_\Sigma) \subset \leftrightarrow_{R_2}^* \cap (T_\Sigma \times T_\Sigma)$ ,
- (ii)  $\leftrightarrow_{R_2}^* \cap (T_\Sigma \times T_\Sigma) \subset \leftrightarrow_{R_1}^* \cap (T_\Sigma \times T_\Sigma)$ ,
- (iii)  $\leftrightarrow_{R_1}^* \cap (T_\Sigma \times T_\Sigma) = \leftrightarrow_{R_2}^* \cap (T_\Sigma \times T_\Sigma)$ ,
- (iv)  $\leftrightarrow_{R_1}^* \cap (T_\Sigma \times T_\Sigma) \not\leftrightarrow \leftrightarrow_{R_2}^* \cap (T_\Sigma \times T_\Sigma)$ ,

where “ $\not\leftrightarrow$ ” stands for the incomparability relationship.

(10) Let  $R$  be a term rewrite system over  $\Sigma$  such that  $R \cup R^{-1}$  is an lgsms term rewrite system. Moreover, let  $g \in \Sigma - \Sigma_0$  be such that  $g$  does not occur in any rule of  $R$ , and let  $\# \in \Sigma_0$  be irreducible for  $R \cup R^{-1}$ . Then it is decidable whether  $R$  is two-way ground minimal. (A term rewrite system  $R$  over  $\Sigma$  is two-way ground minimal if for each rule  $l \rightarrow r$  in  $R$ ,  $\leftrightarrow_{R-\{l \rightarrow r\}}^* \cap (T_\Sigma \times T_\Sigma) \subset \leftrightarrow_R^* \cap (T_\Sigma \times T_\Sigma)$ .)

(11) Let  $R$  be a term rewrite system over  $\Sigma$  effectively preserving recognizability, and let  $p, q \in T_\Sigma(X)$ . Then it is decidable if there exists a tree  $r \in T_\Sigma(X)$  such that  $p \rightarrow_R^* r$  and  $q \rightarrow_R^* r$ .

(12) Let  $R$  be a term rewrite system over  $\Sigma$  effectively preserving recognizability. Then it is decidable if  $R$  is locally confluent.

By direct inspection we obtained that for any deterministic top-down tree transducer  $\mathcal{A} = (\Sigma, \Delta, A, a_0, R)$  with  $\Sigma \cap \Delta = \emptyset$ ,  $R$  is a convergent left-linear gsm term rewrite system over the ranked alphabet  $A \cup \Sigma \cup \Delta$ . Hence Fülöp's undecidability results on deterministic top-down tree transducers simply imply the following. Each of the following questions is undecidable for any convergent left-linear gsm term rewrite systems  $R_1$  and  $R_2$  over a ranked alphabet  $\Omega$ , for any recognizable tree language  $L \subseteq T_\Omega$  given by a tree automaton over  $\Omega$  recognizing  $L$ , where  $\Gamma$  is the smallest ranked alphabet for which  $R_1(L) \subseteq T_\Gamma$ . (Here  $R_1(L)$  is the set of ground  $R_1$ -normal forms of  $L$ , i.e.  $R_1(L) = R_1^*(L) \cap IRR(R_1)$ .)

- (i) Is  $R_1(L) \cap R_2(L)$  empty?
- (ii) Is  $R_1(L) \cap R_2(L)$  infinite?
- (iii) Is  $R_1(L) \cap R_2(L)$  recognizable?
- (iv) Is  $T_\Gamma - R_1(L)$  empty?
- (v) Is  $T_\Gamma - R_1(L)$  infinite?
- (vi) Is  $T_\Gamma - R_1(L)$  recognizable?
- (vii) Is  $R_1(L)$  recognizable?
- (viii) Is  $R_1(L) = R_2(L)$ ?
- (ix) Is  $R_1(L) \subseteq R_2(L)$ ?

Fülöp and Gyenizse showed that it is undecidable for a tree function induced by a deterministic homomorphism if it is injective. Hence the following holds. Let  $R$  be a convergent left-linear gsm term rewrite system over  $\Sigma$ . Let  $L \subseteq T_\Sigma$  be a recognizable tree language. Then it is undecidable if the tree function  $\rightarrow_R^* \cap (L \times R(L))$  is injective.

We say that a term rewrite system  $R$  is collapse-free if there is no rule  $l \rightarrow r$  in  $R$  such that  $l \in X$  or  $r \in X$ . Finally, we [3] showed that preserving recognizability and effectively preserving recognizability are modular properties of linear collapse-free term rewrite systems. That is, the following results hold. Let  $R$  and  $S$  be linear collapse-free term rewrite systems over disjoint ranked alphabets. Then  $R$  and  $S$  preserve recognizability if and only if the disjoint union  $R \oplus S$  of  $R$  and  $S$  also preserves recognizability. Moreover,  $R$  and  $S$  effectively preserve recognizability if and only if the disjoint union  $R \oplus S$  of  $R$  and  $S$  also effectively preserves recognizability. These results imply that preserving recognizability and effectively preserving recognizability are modular properties of  $\lambda$ -free string term rewrite systems.

Let  $R$  be a term rewrite system over  $sign(R)$ , and let  $\Sigma = \{f, \#\} \cup sign(R)$ , where  $f \in \Sigma_2 - sign(R)$  and  $\# \in \Sigma_0 - sign(R)$ . We [3] showed that  $R$  effectively preserves  $\Sigma$ -recognizability if and only if  $R$  effectively preserves recognizability. We [4] improved this result for left-linear term rewrite systems. We showed the following. Let  $R$  be a left-linear term rewrite system over  $sign(R)$ , and let  $\Sigma = \{g, \#\} \cup sign(R)$ , where  $g \in \Sigma_1 - sign(R)$  and  $\# \in \Sigma_0 - sign(R)$ . Then  $R$  effectively preserves  $\Sigma$ -recognizability if and only if  $R$  effectively preserves recognizability. This result makes it easier to show that a given left-linear term rewrite system effectively preserves recognizability. Furthermore,  $R$  preserves  $\Sigma$ -recognizability if and only if  $R$  preserves recognizability.

We [6] introduced and studied the half-monadic term rewrite system, which is an extension of the right-ground term rewrite system and is a slight extension of the semi-monadic term rewrite system. Takai, Kaji, and Seki introduced the concept of the finite path overlapping term rewrite system, and showed that right-linear finite path overlapping term rewrite systems effectively preserve recognizability. A half-



monadic term rewrite system is a finite path overlapping term rewrite system as well. Hence right-linear half-monadic term rewrite systems effectively preserve recognizability. We [6] showed that termination and convergence are decidable properties for right-linear half-monadic term rewrite systems.

## II. GROUND TERM REWRITE SYSTEMS

In paper [5], we showed that a reduced ground term rewrite system over some ranked alphabet  $\Sigma$  has a good property: if we omit any rules, the omitted rules and the remaining rules generate congruence relations such that their intersection is the identity relation on  $T_\Sigma$ . In general, given two ground term rewrite systems  $A$  and  $B$  over some ranked alphabet  $\Sigma$  with  $\leftrightarrow_A^* \subseteq \leftrightarrow_B^*$ , one may want to eliminate all nontrivial pairs of  $\leftrightarrow_A^*$  from  $\leftrightarrow_B^*$ . In other words, one may want to construct a ground term rewrite system (gtrs for short)  $C$  over  $\Sigma$  such that  $\leftrightarrow_{A \cup C}^* = \leftrightarrow_B^*$  and that  $\leftrightarrow_A^* \cap \leftrightarrow_C^*$  is the identity relation on  $T_\Sigma$ . Let  $A$  and  $B$  be gtrs's over some ranked alphabet  $\Sigma$  with  $\leftrightarrow_A^* \subseteq \leftrightarrow_B^*$ . We say that a gtrs  $C$  over  $\Sigma$  is a congruential complement of  $A$  for  $B$ , if  $\leftrightarrow_{A \cup C}^* = \leftrightarrow_B^*$  and  $\leftrightarrow_A^* \cap \leftrightarrow_C^*$  is the identity relation over  $T_\Sigma$ . We note that  $A$  may have more than one congruential complement for  $B$ , which are pairwise nonequivalent. Consider the following example. Let  $\Sigma = \{a, b, f\}$ , where  $a, b$  are of rank 0, and  $f$  is of rank 1. Consider the gtrs's

$$\begin{aligned} A &= \{f(a) \rightarrow a, f(b) \rightarrow b\}, \\ B &= \{f(a) \rightarrow a, f(b) \rightarrow b, a \rightarrow b\}, \\ C &= \{a \rightarrow b\}, \\ D &= \{a \rightarrow f(b)\}, \\ E &= \{f(a) \rightarrow f^3(b)\} \end{aligned}$$

over  $\Sigma$ . Then each one of  $C$ ,  $D$ , and  $E$  is a congruential complement of  $A$  for  $B$ . Moreover,  $\leftrightarrow_C^* \neq \leftrightarrow_D^*$ ,  $\leftrightarrow_C^* \neq \leftrightarrow_E^*$ , and  $\leftrightarrow_D^* \neq \leftrightarrow_E^*$ .

Our [5] main result is the following. Given gtrs's  $A, B, C$  over some ranked alphabet  $\Sigma$  with  $\leftrightarrow_A^* \subseteq \leftrightarrow_B^*$ , one can effectively decide if  $C$  is a congruential complement of  $A$  for  $B$ .

Dauchet et al. have introduced the notion of the ground tree transducer as a pair  $(A, B)$  of tree automata. The importance of ground tree transducers is in that they can simulate ground term rewriting: it was shown that for each ground term rewriting system  $R$  over a ranked alphabet  $\Sigma$ , one can effectively construct a ground tree transducer  $(A, B)$  over  $\Sigma$  such that  $\rightarrow_R^*$  is equal to the tree transformation  $\tau_{(A, B)}$  induced by  $(A, B)$ . Dauchet et al. have used this result to show that the confluence property of ground term rewrite systems is decidable.

In paper [2] we considered restricted versions of ground tree transducers. Our motivation was that studying restricted versions of a class of machines frequently gives a deeper insight into the working of the unrestricted class.

As usual for a class of tree transducers, we considered the total and the deterministic subclasses. We call a ground tree transducer  $(A, B)$  deterministic (total) if the tree automata  $A$  and  $B$  are deterministic (total). We also considered symmetric ground tree transducers, which are of the form  $(A, A)$ . Moreover, we considered the eight classes of ground tree transducers obtained by combining these three properties in all possible ways.

As the first result, we compared the expressive power of the eight classes by presenting the full inclusion diagram of the tree transformation classes induced by them.

Then we [2] showed that the following four classes of term relations are the same: (i) the tree transformations induced by symmetric deterministic ground tree transducers, (ii) the congruence relations on term algebras induced by reduced ground term rewriting systems, (iii) the congruence relations on term algebras induced by convergent ground term rewriting systems, and (iv) the finitely generated congruence relations on term algebras.

Fülöp and Vágvölgyi showed the following. For every ground term equation system  $E$  over a ranked alphabet  $\Sigma$ , which is just a finite binary relation on  $T_\Sigma$ , one can effectively construct a deterministic tree automaton  $A$  over  $\Sigma$  such that  $\leftrightarrow_E^*$ , i.e., the congruence relation induced by  $E$  on the  $\Sigma$ -term algebra, is equal to the tree transformation  $\tau_{(A, A)}$  induced by the symmetric deterministic ground tree transducer  $(A, A)$ .

In the proof of the inclusion (i)  $\subseteq$  (ii) we constructed, for a given deterministic tree automaton  $A$  over a ranked alphabet  $\Sigma$ , a reduced ground term rewriting system  $R$  over  $\Sigma$  such that the congruence relation  $\leftrightarrow_R^*$  generated by  $R$  is equal to the tree transformation  $\tau_{(A, A)}$ .

Thus, as a by-product of our results, we obtained a new ground completion algorithm. Given ground term equation system  $E$ , we constructed a reduced ground term rewriting system equivalent to  $E$  in two steps. In the first step, we computed a symmetric deterministic ground tree transducer  $(A, A)$  such that  $\tau_{(A, A)} = \leftrightarrow_E^*$ . Then we constructed as in the proof of (i)  $\subseteq$  (ii) a reduced ground term rewriting system  $R$  such that  $\tau_{(A, A)} = \leftrightarrow_R^*$ . Hence  $\leftrightarrow_E^* = \leftrightarrow_R^*$ . This ground completion parallels to Snyder's fast algorithm.

It can easily be proved that a binary relation  $\tau$  over  $T_\Sigma$  can be induced by a total symmetric deterministic ground tree transducer if and only if  $\tau$  is a congruence of finite index on the  $\Sigma$ -term algebra. In addition, we [2] showed that any finitely generated congruence relation  $\tau$  over the  $\Sigma$ -term algebra is of finite index if  $\text{trunk}(\tau) = T_\Sigma$ . The concept of the trunk was introduced and proved to be useful in studying congruence relations over terms. Furthermore, we proved that for a term algebra with at least one non-nullary function symbol the congruence relations of finite index are exactly the finitely generated congruence relations with trunks equal to the set of all terms.

In paper [1] we studied that version of the ground tree transducer game where the same tree automaton appears as the first and second component of the associated ground tree transducer. We gave conditions which imply that Beta has a winning strategy. Furthermore, we showed the following decidability result. Given a ground tree transducer game where the underlying tree automaton  $A$  cannot evaluate some tree into a state or  $A$  is deterministic, we can decide which player has a winning strategy. Moreover, whatever player has a winning strategy, we can effectively construct a partial recursive winning strategy for him.

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